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Dynamic role of the liquid core of Mercury in its motion on Cassini's laws and in resonant librations

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Introduction. New approaches to the study of Mercury dynamics and the construction of analytical theory of its resonant rotation are suggested. Within these approaches Mercury is considered as a system of two non-spherical interacting bodies: a core and a mantle. The mantle of Mercury is considered as non-spherical, rigid (or elastic) layer. Inner shell is a liquid core, which occupies a large ellipsoidal cavity of Mercury. This Mercury system moves in the gravitational field of the Sun in resonant traslatory-rotary regime $3n = 2\omega$ (n is the mean orbital motion and ω is the rotational angular velocity) [1]. In considered model Mercury moves on elliptical precessing orbit with inclination to Laplace plane $i = 7^0 0029$ and with eccentricity$e = 0.2056$. The orbit plane precesses with respect to normal $\mathrm{n_L}$ to Laplace plane with the small angular velocity $n_\Omega < 0$, executing the return (regressive) motion. We have $n_\Omega/n = -0.8294\cdot 10^{-6}$. For the study of Mercury rotation we have been used specially designed author's canonical equations of motion in Andoyer and Poincare variables [2].

Mercury model. On the first stage of study we have evaluate the gravitational parameters of Mercury. For this we have used last determinations of the amplitude of Mercury libration in longitude $53"2 \pm 2"0$and mean angle $\rho_0 = 2'11 \pm 0'10$ between the axis of Mercury rotation and normal to the orbital plane $\mathrm{n_L}$. These very exact values of parameters of Mercury rotation have been obtained on the base of ground radio-location observations [3]. On the base of theoretical relation between mentioned parameters and parameters of Mercury gravitational field $J_2$and C_{22}, presenting analytical expression of the fundamental Cassini's low (see (1)), and analytical expression of amplitude of libration of two layer Mercury (with a liquid core) we have obtained following evaluations of parameters $J_2 = (4.23 \pm 0.06) \cdot 10^{-5}$,

$C_{22} = (0.85 \pm 0.05) \cdot 10^{-5}$. They can be compared with values which were obtained earlier from observational data of Mariner-10 mission: $J_2 = (6 \pm 2) \cdot 10^{-5}$and $C_{22} = (1.0 \pm 0.5) \cdot 10^{-5}$[4]. Also we have used here theoretical values of dimensionless moment of inertia$I = C/(mR^2) = 0.34$and $C_m/C = 0.5 \pm 0.07$ [5]. Here C and C_m are the polar moments of inertia of the full Mercury and of its mantle; $C_c = C - C_m$ is the polar moment of inertia of the core; m and R is the mass and the mean radius of Mercury. We have used here an additional assumption that oblatenesses of the ellipsoids of inertia of the mantle and the full Mercury approximately equal. In result we have obtain the following model of the tensor of inertia of the full Mercury and its mantle: $A = 0.3399407 \cdot mr^2$, $B = 0.34 \cdot mr^2$, $C = 0.3399747 \cdot mr^2$, $A_m = A_c = 0.1699703 \cdot mr^2$,$B_m = B_c = 0.17 \cdot mr^2$, $C_m = C_c = 0.1699874 \cdot mr^2$.

Cassini's laws for two-layer resonant planet. For rigid model of Mercury the generalized Cassini's laws was obtained by G. Colombo [1] and then studied by others scientists (Peale, 1969; Beletskii, 1972; Ward, 1975 and oth.) including consideration the tidal deformations. In the first these laws have been obtained and formulated for model of the Moon with liquid core and Mercury with liquid core by authors [2]. In our paper we have used special canonical and non-canonical forms of rotation of the planet with ideal liquid core which occupies the ellipsoidal cavity [2]. Here we give final formulations of the laws in detailed form: 1. Vectors of angular velocities and angular momentums of the liquid core and Mercury coincide with its polar axis of inertia. 2. The mantle-core system of Mercury rotates as one rigid body about polar axis of inertia in direction of its orbital motion with constant angular velocity equal 3/2 from the mean orbital motion of Mercury n with respect to geocentric ecliptic reference system connected and rotated with mean apsidal line in the orbit plane. 3. Every passage of perigee the equatorial axes of minimal moment of inertia of Mercury is oriented towards to the Sun centre consequently changing own orientation on opposite. In apogee this axis is oriented on tangential axis to elliptic orbit and the axis of the middle moment of inertia is oriented to the Sun centre. These axes also change own orientation by passage of apogee on opposite every orbital period. 4. Mean ascending node of the Mercury orbit on Laplace plane coincides with the mean ascending node of equator of Mercury figure (or of the plane orthogonal to vectors of angular momentums of the core $\mathrm{G_c}$and Mercury G). The general node of mentioned planes makes return motion along Laplace plane with constant angular velocity $|n_\Omega|$. The vectors $\mathrm{G_c}$, G and normal to Laplace plane $\mathrm{n_L}$ and normal to the mean orbit plane n are situated in the plane orthogonal to the line of node of orbit on the Laplace plane and form the constant angle with each other. 5. Angular momentums of Mercury and its core make a constant small angle ρ_0 with the normal to orbit plane which depends from dynamical oblatenesses of Mercury, from a precession velocity of Mercury orbit plane and from inclination i and eccentricity e of its orbit:

$$\rho_0 = -\sin i \left[\cos i + n \left(J_2 X_0^{(-3.0)} + 2C_{22} X_3^{(-3.2)} \right) \Big/ (I n_\Omega) \right]^{-1} \qquad (1)$$

where $X_0^{(-3.0)}$ and $X_3^{(-3.2)}$ are known eccentricity functions. For accepted values of Mercury's parameters and its eccentricity $e = 0.2056$(numerical values$X_0^{(-3.0)} =$ 1.0669439, $X_3^{-3.2} =$ 0.654214) the equation (1) determines the observed value $\rho_0 = 2'11$.

Forced librations. Forced librations of Mercury in longitude have been studied on the base of the model of plane motion [6]. The amplitude of annual libration in considered study consists $35"3 \pm 2"0$ (period 87.97 days). The semiannual libration is characterized by amplitude $3"72 \pm 0"24$ (with opposite phase; period 43.98 days). A phenomenon of non - perturbance of Mercury rotation in vicinity of pericenter of orbit has been established. In period of 15 days the angular velocity of Mercury saves almost the permanent value.

Resonant librations. On the next step we have evaluated frequencies and periods of forced resonant oscillations of core-mantle system of Mercury on the base of specially constructed analytical theory. So period of libration in longitude consistsT_g=12.2$\pm$0.3 yr. Period of the pole wobble of Mercury is equal $T_l = 426 \pm 25$ yr. The period o