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The significance of degrees of freedom when using chaos theory methodologies for the prediction of radar rainfall

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At high resolution, the development and dissipation of extreme storm events over small localised areas show very little of the seasonal trends that can be utilised for synoptic forecasting, in many cases appearing random in nature. Due to this characteristic, a chaos theory technique known as the method of delays has been adopted in this paper over classical signal processing, which often require seasonal trends, making the chaotic assumption that the studied rainfall events appear random, but evolve from a deterministic system.

In traditional chaotic time series analysis, the well known Takens' theorem is used as a method of delays to reconstruct the underlying dynamics of a system, using information from the time series of one observable linked to the system (Takens, 1981). In order to incorporate information from neighbouring locations from radar rainfall to our phase space reconstruction, we are no longer looking at a single time series but a spatially extended system. Consequently, by adapting Takens' methodology here to a spatially extended series a further assumption must be applied, which states we are working at a high enough resolution whereby surrounding locations are driven by similar dynamics. With this view it is hoped that the inherent assumptions of the time series approach, which do not easily transfer to spatial extension, can be considered to loosely hold.

This paper aims to determine the importance of parameter precision for each location within a localised neighbourhood during a rainfall event, and in so doing identify how this importance is affected by storm type and intensity. The two main parameters in the method of delays is the delay increment α, and the embedding dimension d, which were approximated using the minimum mutual information criterion (Abarbanel, 2001) and correlation dimension (Grassberger and Procaccia, 1983) respectively.

For the first approach, information from surrounding locations is constrained to the structural parameters of the location of interest s, producing the following structure for a three location neighbourhood state vector, where i represents the time index of the rainfall observable x:

$$X_i^s = \Big(x_i^{s-2}, x_{i-\alpha_s}^{s-2}, x_{i-2\alpha_s}^{s-2}.....x_{i-(d_s-1)\alpha_s}^{s-2}, x_i^{s-1}, x_{i-\alpha_s}^{s-1}, x_{i-2\alpha_s}^{s-1}$$

$$.....x_{i-(d_s-1)\alpha_s}^{s-1}, x_i^s, x_{i-\alpha_s}^s, x_{i-2\alpha_s}^s.....x_{i-(d_s-1)\alpha_s}^s\Big)$$

This constrained technique is then compared using a local linear model for prediction, to those where each individual location keeps as much of its individual characteristics as possible, giving the following structure for a three location neighbourhood state vector:

$$X_i^s = \Big(x_i^{s-2}, x_{i-\alpha_{s-2}}^{s-2}, x_{i-2\alpha_{s-2}}^{s-2}.....x_{i-(d_{s-2}-1)\alpha_{s-2}}^{s-2}, x_i^{s-1}, x_{i-\alpha_{s-1}}^{s-1}, x_{i-2\alpha_{s-1}}^{s-1}$$

$$.....x_{i-(d_{s-1}-1)\alpha_{s-1}}^{s-1}, x_i^s, x_{i-\alpha_s}^s, x_{i-2\alpha_s}^s.....x_{i-(d_s-1)\alpha_s}^s\Big)$$

Radar rainfall data has been used from the UK Meteorological Office's Nimrod System with time resolution of 5 minutes and spatial resolution of 2km. Preliminary results have shown that during heavy rainfall, adjacent neighbours to s give structure parameter combinations that vary by only a few degrees, with differences increasing as you move away to other areas of the rainfall event as you might expect. Although this may suggest a preference to the first approach, reducing computational time, there is also the need to extend the reconstruction dimensions, to be sure of incorporating as much information from the original underlying dynamics as computationally possible. Combining this with the sensitive dependence on initial conditions fundamental to chaos, the second approach becomes more suitable to the storm evolution and may produce longer predictive periods.

From these results it appears it may be possible to construct a rough idea of the spatial structure of a historic rainfall event, which may help to further our understanding of the intricacies taking place during storm events.

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