



Assessment of multifractal spectra of river networks

Leonardo Primavera (1), Ana M. Tarquis (2), Samuele De Bartolo (3), Roberto Gaudio (3), Massimo Veltri (3)

1. Dipartimento di Fisica, Università della Calabria, Ponte P. Bucci, Cubo 33/b, 87036 – Rende (CS), Italy
2. Departamento de Matemática Aplicada a la Ingeniería Agronómica. Escuela Técnica Superior de Ingenieros Agrónomos, Universidad Politécnica de Madrid, Spain
3. Dipartimento di Difesa del Suolo “V. Marone”, Università della Calabria, Ponte P. Bucci, Cubo 42/b, 87036 – Rende (CS), Italy

During time, the multifractal geometry has got increasing relevance as a tool for understanding many complex phenomena. In fluvial geomorphology, as a particular case, the research carried out in the last decades has evidenced that the scaling properties of drainage basins, river networks and braided channels are better understood in the framework of multiscaling and multifractal, instead of simple fractal sets.

In particular, it was evidenced by several works that two-dimensional projections of river networks and braided channels on topographic maps are multifractal, non-plane-filling structures (support fractal dimension less than two). Moreover, the multifractal parameters obtained from the analysis can be used for a redefinition of the Instantaneous Unit Hydrograph (IUH), in order to improve the prediction of the hydrologic response of a basin (flood hydrograph).

In order to get definite conclusions about such matters, however, a fundamental task to be accomplished is the precise assessment of both positive and negative moment orders of probabilities of the multifractal spectra. The determination of the complete multifractal spectra, when carried out on natural objects, like rivers and channels, has to be performed through numerical methods. Traditionally, two separated classes of numerical algorithms exist, which lead to the assessment of the multifractal spectra

of a given set of points: Fixed-Size Algorithms (FSAs) or Fixed-Mass Algorithms (FMAs). Methods belonging to the former class are: 1) standard box-counting algorithms (Block et al., 1990); 2) the sandbox method (Tél et al., 1989); 3) the Generalized Correlation Integral Method (GCIM) (Pawelzik and Schuster, 1989); 4) the Gliding Boxes Algorithm (GBA) (Allain and Cloitre, 1991). Concerning the latter class, the standard FMA by Badii and Politi (1984, 1985), was applied in the past (De Bartolo et al., 2006).

The performed analyses have shown that, in general, the fixed-size standard box-counting methods suffer both “border effects” and difficulties in reconstructing the right side of the multifractal spectrum (corresponding to the negative order moments). “Border effects” are due to the presence of boxes overlaying the edges of the basin, when the set of points representing the river network is covered with meshes of decreasing size. The difficulties in estimating correctly the negative moment orders are due to the presence of boxes with few points (namely with a low statistics for the determination of the moments). Such problems are partially overcome when using the sandbox method or the Correlation Integral Method, among the fixed-size algorithms. Usually, the GBA method gives good results for negative moment orders, in the case of large sample size. Finally, the standard fixed mass algorithm yields much better results in overcoming both problems, by assessing the multifractal spectrum of studied river networks with reduced error bars.

In the present lecture, we show several comparisons among different algorithms implemented in the recent years, concerning the determination of the multifractal spectra of river networks and braided channels, which confirm the multifractal nature of the analyzed systems and the singularity indices of the Lipschitz-Hölder useful for the definition of flood routing models.

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