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Estimators of Long-Memory: a review of recent advances

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We study here finite variance stochastic processes $\{X_k\}_{k\geq 1}$, whose spectral density $f(\lambda)$, $\lambda \in (-\pi, \pi)$ behaves like a power function at low frequencies, that is as $|\lambda|^{-2d}$ as the frequency $\lambda \to 0+$. The case $d > 0$ corresponds to *long-memory*, $d = 0$ to *short-memory* and $d < 0$ is often referred to as *negative dependence*. For $X_k, k \in$ to be stationary it is necessary that $\int_{-\pi}^{\pi} f(\lambda) d\lambda < \infty$ and hence that $d < 1/2$. We relax these restrictions in a number of ways. We shall allow the process to be non-stationary, requiring only that it becomes stationary after it is differenced a number of times. We also suppose that the spectral density (of the differenced process) behaves not merely like $|\lambda|^{-2d}$ but as $|\lambda|^{-2d} f^\star(\lambda)$, where $f^\star$ is regarded as a short-range density function.

Our goal is to estimate d in the presence of $f^\star$. We shall not assume that the nuisance function $f^\star$ is known, nor that it is characterized by a finite number of unknown parameters, but merely that $f^\star(\lambda)$ is "smooth" in the neighborhood of $\lambda = 0$, so that if one focuses only on frequencies λ that are sufficiently low, then the spectral density $f(\lambda)$ behaves essentially like $|\lambda|^{-2d}$. What frequency cut-off should one choose will clearly become an important issue.

The estimation framework is *semi-parametric*: we must estimate the unknown parameter d while viewing the presence of $f^\star$ as a nuisance, albeit one which complicates matters. The estimation method will also be *local*, in that, it is necessary to focus only on frequencies λ that are close enough to the origin, where the influence of $f^\star(\lambda)$ can be neglected.

In this paper we provide an overview and comparison of four semi-parametric estima-

tion methods of the parameter d which have all proven to be very effective. Two of them are Fourier-based, the other two are based on wavelets. The methods are:

- Geweke-Porter Hudak (GPH): Regression / Fourier,
- Local Whittle Fourier (LWF): Whittle / Fourier,
- Local Regression Wavelets (LRW): Regression / Wavelets,
- Local Whittle Wavelets (LWW): Whittle / Wavelets.

The Fourier methods are older and better known. They have essentially been developed by Peter Robinson in a number of fundamental papers. If we ignore for the moment the presence of the nuisance function $f^\star$, then one has $f(\lambda) = |\lambda|^{-2d}$, that is $\log f(\lambda) \approx -2d \log |\lambda|$. Therefore, d can be estimated by linear regression on the periodogram. This is the Fourier-based regression method considered in Gewek and Porter-Hudak (1983) in a parametric setting. The semi-parametric setting was suggested by Künsch (1987) and developed later by Robinson (1995). The Fourier-based Whittle method is a pseudo-maximum likelihood method developed by Fow and Taqqu (1986) in a parametric setting and extended in a semi-parametric setting by Robinson (1995).

The papers of Moulines, Roueff and Taqqu (2005-2007) recast the preceding Fourier-based methods in a wavelet setting. Wavelets have a number of advantages. They allow differencing implicitly and therefore they can be used without problems when $d > 1/2$. They also **automatically** discount polynomial trends. The local wavelet-based regression method was first developed by Abry and Veitch (1998) under the simplifying assumption that the wavelet coefficients are uncorrelated. The Local Whittle wavelet method was developed in Moulines, Roueff and Taqqu (2007).

We will show how these methods can be used to develop test procedure of long-memory against various alternatives, including jump in the mean and in the variance.