



Stochastic climate modeling and random dynamical systems: robustness, frequency locking and applications

M.Ghil (1,2), M.D.Chekroun (2), E.Simonnet (3)

(1) LMD-IPSL, CNRS, ENS, F-75231 Paris Cedex 05, France, and Dep. of Atmos. and Oceanic Sciences and IGPP, UCLA, Los Angeles, CA 90095-1567, USA. (2) Environmental Research and Teaching Institute, ENS, Paris. (3) Institut Non Lineaire de Nice, UMR 6618, CNRS, 1361 route des Lucioles, F06560 Valbonne, France.

In the last few years, stochastic climate modeling has become a major research thrust in climate dynamics and the geosciences in general. *A priori* insights into the effect of stochastic parametrizations on model dynamics are, however, difficult to obtain. This is especially true with respect to the model's long-term, asymptotic behavior and sensitivity to parameters. For instance, the lack of robust and efficient parametrizations for general circulation models (GCMs) seems to present a major obstacle in reducing uncertainties in climate-change projections for the end of the century.

Independently, the theory of random dynamical systems (RDS) developed by L. Arnold and associates has tried to merge the concepts and tools of stochastic processes and deterministic dynamical systems. RDS theory thus offers a convenient mathematical framework to study in depth the effect of stochastic parametrizations on a hierarchy of models.

Using the tools of RDS theory, we consider the effect of noise on the so-called V. I. Arnol'd family of maps, numerically as well as theoretically. We show that the noisy Arnol'd map family exhibits two important and inter-related phenomena: (i) the selective disappearance of the weaker locked frequencies and narrowing of the remaining, more robust ones; as well as (ii) the narrowing of the parameter range within which frequency-locking occurs. We also study rigorously the model's robustness via the concept of *stochastic structural stability*.

Our theoretical results seem to provide a tentative explanation for numerically obtained results on robustness, with respect to small parameter changes, of certain stochastic models in the El-Niño/Southern-Oscillation (ENSO) literature. For instance, several fully deterministic ENSO models of various degrees of complexity yield a reasonable spatial structure and spatio-temporal variability. Some of these models, however, exhibit too strong a frequency locking, i.e., too many possible resonances between the internal ENSO mode and the seasonal cycle or, equivalently, too many steps on the Devil's staircase generated by the interaction between the annual and internal frequency. In the presence of noise, certain models do exhibit the disappearance of some of these steps or even the total disappearance of frequency locking, as apparent in our study. The most robust steps are those that seem to be found in both observations and full-fledged coupled GCMs. The present approach and results thus seem to show how RDS theory could be useful in the study of stochastic climate models.