Geophysical Research Abstracts,
Vol. 10, EGU2008-A-09460, 2008
SRef-ID: 1607-7962/gra/EGU2008-A-09460
EGU General Assembly 2008


# Scalable robust solvers for 3D unstructured modeling applications, solving the Stokes equation with large, localized, viscosity contrasts.

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The solution of the Stokes equation in mantle convection applications has always been the main time consuming operation. The transition from 2D to 3D has only worsened this situation due to the sub optimal scaling of popular solver implementations.

To be able to solve high resolution 3D convection problems, we have to use scalable solvers both with respect to the number of degrees of freedom as well as having optimal parallel scaling characteristics on computer clusters

Geometric multigrid (GMG) type methods satisfy these criteria but suffer from erratic robustness characteristics and constraints with respect to the geometry and the topology of the model domain. If material properties are distributed in an unfavorable way the method will break down resulting in poor convergence.

For isoviscous models GMG is arguably the fastest solution method for problems with over 10 million degrees of freedom. However for orders of magnitude viscosity contrasts GMG has the tendency to break down. There are two reasons why GMG might break down that are related to the interpolation between coarse grids and the construction of these grids.

The first issue was first addressed by Alcouffe in 1981. His solution comes down to using the continuity of $\eta\nabla\mathbf{u}$ rather than $\nabla\mathbf{u}$ for the interpolation between grids, that basically means using Algebraic Multigrid (AMG) for the interpolation between grids and GMG for the coarse grid construction. This is an effective way of dealing with strong jumps across internal boundaries. However this method can still break down when contrast in viscosity occur at the finest resolution. Localization effects are a typical example. In such cases the construction of the coarse grids should also reflect the distribution of viscosity contrasts and hence should be constructed based on the fine grid operator. Another advantage of AMG type solvers/preconditioner as opposed to GMG is there robustness for arbitrary geometry and topology, allowing for unstructured grids.

We present an implementation of AMG used as a preconditioner for the solution of the Stokes equation with a Krylov solver (CG) on an unstructured grid. This implementation has the same scaling characteristics as GMG, linear scaling with the number of degrees of freedom and number of Cpu's, but is robust for fine scale, large viscosity contrasts associated with localization phenomena.