



A simplified analytical model of the probabilistic dynamics of soil nitrate

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The importance of the relationships between ecosystem nutrient availability and eco-hydrological processes is evidenced by the growing attention from hydrologists and ecologists. A full understanding of the variety of connections and feedbacks existing between key hydrologic fluxes and the various components of soil nutrient cycles, however, is still lacking. In particular, the specific role of climate and bio-geochemical processes in causing nutrient limitation in ecosystems is still poorly understood. In this contribution we analyze the probabilistic dynamics resulting from a simplified model of soil moisture and nitrate mass in relatively arid environments, which accounts for relevant hydrologic and bio-geochemical processes and for the random characters of rainfall. The soil-nitrate balance includes uptake through transpiration, mineralization, nitrification, and denitrification. To allow an exact mathematical treatment, all nitrate fluxes are assumed to be linear functions of the state variables, namely soil moisture and nitrate mass stored in soils. The range of applicability of the above simplification is investigated by means of numerical Monte Carlo simulations, showing that the linear approach is meaningful in relatively arid environments, where soil moisture contents are generally low. The moment generating function of the joint probability distribution (pdf) of soil nitrate and water content is derived, thereby allowing a linkage between the main features of soil nitrate statistics and the underlying soil, vegetation, rainfall, and bio-geochemical parameters. Exact expressions for the moments

of the nitrate pdf and for the covariance of soil moisture and nitrate mass are derived. The pdf of the nitrate mass in storage within the soil is shown to be reasonably approximated by a gamma distribution in many cases of practical interest. Interestingly, nitrogen limitation for the ecosystem is shown to be directly related to low values of the product between the rate of rainfall arrivals and the characteristic time of nitrate removal.