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Application of special types of nonnegative matrices to forest ecosystems modelling

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The classical Leslie model for age-structured population dynamics (1945) had limited application because of its rigid assumptions, which confined the pattern of nonzero entries in the projection matrix to the first row and subdiagonal only. Twenty years later, L.P. Lefkovitch (1965) attenuated the rigidity of age classes with ‘stages’ of flexible duration, thus having expanded the nonzero pattern with the principal diagonal. The subsequent boom of applications in population biology has revealed that the ‘stages’, as the basis to classify individuals, can be understood in a broader sense as stages of development, or (intervals of) the body size, or anything else from the biology of individuals that enables construction of the *life cycle graph* (Caswell, 2001).

However, both chronological age and ontogenetic stages are of explicit importance in applications to extensive growth of colonizing species (Logofet et al. 2006). Although double bases for classification, like, e.g., age and size (Law 1983), were known since last 80s, the published formalism was too tedious for application in our special cases. Combining age and an additional demographic classification resulted in block-structured projection matrices: the macropattern coincided with a Leslie matrix, while its formerly scalar entries were now replaced with certain blocks. Matrix size increased drastically, which did not facilitate broad applications. If we applied the block-structured formalism in our special case of age and ontogenetic stage, it would be too excessive, resulting in many idle states. Instead, concatenating the complete "matrix" of all virtual states into a (column) vector of only feasible age-stage-specific states reduces the projection matrix back to a non-block formalism, yet with a wider-than-Lefkovitch pattern of nonzeros (Logofet, 2002): the *Logofet* pattern means

obligatory zeros only between the first row and principal diagonal. Classical Perron – Frobenius analysis is still applicable for the *reproductive core* – the major strongly connected component of the life cycle graph; it distinguishes population growth, decline, or equilibrium by whether the dominant eigenvalue is respectively grater, less, or equal to 1.

The above trichotomy can also be resolved via what we call a *potential-growth indicator* (PGI) – a calculable function of demographic parameters with a property that the (in)equality holds true for the dominant eigenvalue if the corresponding (in)equality holds true for the function. Known for the Leslie matrix, an explicit expression of the potential-growth indicator was found for the Lefkovitch one (Logofet and Klochkova, 2002), and the PGI Theorem has been recently expanded to Logofet matrices. Explicit expression for the PGI has appeared

useful in model calibration on a short series of empirical data of forest ecosystems (Logofet at all 2006).

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References

Caswell H., 2001. Matrix population models: construction, analysis, and interpretation. Sinauer Associates, Sunderland, MA.

Law R., 1983. A model for the dynamics of a plant population containing individuals classified by age and size. Ecology 64: 224–230.

Lefkovich L.P., 1965. The study of population growth in organisms grouped by stages. Biometrics 21: 1-18.

Logofet D.O., 2002. Three sources and three constituents of the formalism for a population with discrete age and stage structures. Matematicheskoe Modelirovanie (Mathematical Modelling) 14: 11-22 (in Russian with English summary).

Logofet D.O. and Klochkova I.N. 2002. Mathematics of the Lefkovitch model: the reproductive potential and asymptotic cycles. Matematicheskoe Modelirovanie (Mathematical Modelling) 14: 116–126 (in Russian).

Logofet D.O., Ulanova N. G., Klochkova I. N. and Demidova A. N., 2006. Structure and dynamics of a clonal plant population: classical model results in a non-classic formulation. Ecological Modelling 192: 95-106.