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Wave-activity conservation laws for the three-dimensional anelastic and Boussinesq equations with a horizontally homogeneous background flow

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Wave-activity conservation laws are key to understanding wave propagation in inhomogeneous environments. Their most general formulation follows from the Hamiltonian structure of geophysical fluid dynamics. For large-scale atmospheric dynamics, the Eliassen-Palm wave activity is a well-known example and is central to theoretical analysis. On the mesoscale, while such conservation laws have been worked out in two dimensions, their application to a horizontally homogeneous background flow in three dimensions fails because of a degeneracy created by the absence of a background potential vorticity gradient. Consideration of a background flow depending only on altitude is motivated by the parameterization of subgrid-scales in climate models where there is an imposed separation of horizontal length and time scales. Here we show how this degeneracy can be overcome and wave-activity conservation laws derived for three-dimensional disturbances. Explicit expressions for pseudoenergy and pseudomomentum in the anelastic and Boussinesq cases are derived, and it is shown how the previously derived relations for the two-dimensional anelastic and Boussinesq models can be treated as a limiting case of the three-dimensional problem. The results also generalize earlier three-dimensional results in that there is no slowly varying WKB-type requirement, and the results are extendable to finite amplitude. The relationship $A^{\mathcal{E}} = cA^{\mathcal{P}}$ between pseudoenergy $A^{\mathcal{E}}$ and pseudomomentum $A^{\mathcal{P}}$, where c is the horizontal phase speed, has important applications to gravity-wave parameterization and provides a generalized statement of the first Eliassen-Palm theorem.