



## Multiple scales in deep atmosphere anelastic flows

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The anelastic models of Ogura and Phillips, Dutton and Fichtl, Lipps and Hemler, Durran, or Bannon (see, e.g., Bannon, 1996), filter sound modes while keeping both internal gravity waves and advection. They are attractive for numerical modelling sparing the modeller the handling of numerical stiffness associated with the sound modes. Anelastic models are typically derived through single-scale analyses, i.e., by assuming a single characteristic scale each for time, and the vertical and horizontal extent of the considered flows. The vertical scale, in particular, is the density or pressure scale height.

The breaking internal of gravity waves (see, e.g., Achatz, 2007) induces an important vertical transport of momentum and energy and it occurs at altitudes that are much larger than the pressure or density scale height, and rather comparable to the scale height of potential temperature. Are the anelastic models, originally derived for vertical scales comparable to the density scale height, applicable to this kind of phenomenon? If so, what is the justification?

In this presentation, we revisit the scale analysis of anelastic models using methods of multiple scales asymptotics, [Klein, 2000, 2004]. In particular, we show that (i) only Ogura-Phillips-type single-scale analyses, which assume weak potential temperature stratification comparable to the square of the Mach number, are asymptotically consistent, (ii) the single-scale analyses otherwise neglect a time scale separation between advection and internal waves unless the equation of state is modified so that  $R/c_p$  is also small, and (iii) the processes leading to high-altitude internal wave breaking for realistic stratification and equations of state feature multiple length and time scales

and are not amenable to single-scale analysis. We then report on the state of current efforts at developing such a systematic multiple-scales asymptotic anelastic theory.

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