



Generalized Cassini's motion of Mercury with liquid core and its librations

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Mercury is considered as a system of two non-spherical interacting bodies: the core and the mantle. The mantle of Mercury is considered as non-spherical, rigid layer. Inner shell is a liquid core, which occupies a large ellipsoidal cavity of Mercury. This Mercury system moves in the gravitational field of the Sun in resonant traslatory-rotary regime $3n=2w$ (n is the mean orbital motion and w is the rotational angular velocity) (Colombo, 1965). In considered model Mercury moves on elliptical precessing orbit with inclination to Laplace plane $i=7.0020$ degrees and with eccentricity $e=0.2056$. The orbit plane precesses with respect to normal to Laplace plane with the small angular velocity $p < 0$, executing the regressive motion. We have a evaluation $p/n = -0.8294 \times 10^{-6}$. We take into account only the second harmonic of the force function of the Sun and Mercury. For the study of Mercury rotation we have been used specially designed author's canonical equations of motion in Andoyer and Poincare variables (Barkin, Ferrandiz, 2003, 2004). **Base parameters and Mercury model:** the amplitude of Mercury librations in longitude $35^{\circ}3' \pm 2''0$ and mean value of angle between the axis of Mercury rotation and normal to the orbital plane equal $K = 2^{\circ}11' \pm 0''10$. These sufficiently exact values of parameters of Mercury rotation have been obtained on the base of the ground radar observations (Margot et al., 2007). On the base of theoretical relations between mentioned parameters and parameters of Mercury gravitational field J_2 and C_{22} presenting analytical expression of the fundamental Cassini's law, and analytical expression of amplitude of librations of two layer Mercury (with a liquid core) we have obtained following evaluations of parameters $J_2 = (4.23 \pm 0.06) \times 10^{-5}$ and $C_{22} = (0.85 \pm 0.05) \times 10^{-5}$. However we have

used here fixed theoretical values of dimensionless moment of inertia $I=0.34$ and ratio $C_m/C=0.5$. Here C and C_m are the polar moments of inertia of the full Mercury and of its mantle. We have used here an additional assumption that oblatenesses of the ellipsoids of inertia of the mantle and the full Mercury approximately equal. In result we have obtained a model of the tensor of inertia of the full Mercury and its mantle (Barkin, Ferrandiz, 2003). **Generalized Cassini's laws for two-layer resonant planet:** **1)** Vectors of angular velocities and angular momentums of the liquid core and Mercury coincide with its polar axis of inertia; **2)** The mantle-core system of Mercury rotates as one rigid body about polar axis of inertia in direction of its orbital motion with constant angular velocity ω equal $3/2$ from the mean orbital motion of Mercury n with respect to geocentric ecliptic reference system connected and rotated with mean apsidal line in the orbit plane; **3)** Every passage of perihelion the equatorial axis of minimal moment of inertia of Mercury is oriented towards to the Sun centre consequently changing own orientation on opposite. In aphelion this axis is oriented on tangential axis to elliptic orbit and the axis of the middle moment of inertia is oriented to the Sun centre. These axes also change own orientation by passage of apogee on opposite every orbital period; **4)** Mean ascending node of the Mercury orbit on Laplace plane coincides with the mean ascending node of equator of Mercury figure (or of the plane orthogonal to vectors of angular momentums of the core G_c and Mercury G). The general node of mentioned planes makes return motion along Laplace plane with constant angular velocity p . The vectors G_c , G and normal to Laplace plane and normal to the mean orbit plane are situated in the plane orthogonal to the line of node of orbit on the Laplace plane and form the constant angle with each other; **5)** Angular momentums of Mercury and its core make a constant small angle $K=2'1$ with the normal to orbit plane which depends from dynamical oblatenesses of Mercury, from a precession velocity of Mercury orbit plane and from inclination i and eccentricity e of its orbit. **Forced librations.** Forced librations of Mercury in longitude have been studied on the base of the model of plane motion. The amplitude of annual librations in considered study consists $35''3+/-2''0$ (period 87.97 days). The semiannual librations is characterized by amplitude $3''7+/-0.2$ (with opposite phase; period 43.98 days). A phenomenon of non-perturbation of Mercury rotation in vicinity of perihelion of orbit has been established (Barkin, 1979). In period about 15 days the angular velocity of Mercury saves almost the permanent value. **Resonant librations.** On the next step we have evaluated frequencies and periods of forced resonant oscillations of core-mantle system of Mercury on the base of specially constructed analytical theory. So period of librations in longitude consists $12.3+/-0.3$ yr, period of the pole wobble of Mercury is about $426+/-25$ yr and precession period consists about $1460+/-70$ yr. The liquid core interacts with the mantle and generates additional pole librations with period $T_c=58.6251$ days. The value $T_c=58.62$ days was obtained

earlier (Barkin, Ferrandiz, 2003). The work was accepted by Spanish grants.