



Scaling and multiscaling of soils as heterogeneous complex networks

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A complex network is a set of nodes and links with a non-trivial topology [1]. Networks provide a metaphor for the coupling architecture of interacting components in a complex system. From this standpoint, networks pervade all domains of science: natural [2], social [3], technological [4] and cultural [5].

From this abstraction of complex systems, it is possible to study the soil as a network in which pores are the elements linked by micro “tubes” with others. A soil with big particles like sand displays a pore network highly connected. On the other hand, a clay soil has a sparse network. The pore network is related with density and draining of the soil.

The topology of a soil network can be simulated by dynamical network models [6] as discrete-time dynamical systems that prescribe the evolution of a network by the iterated addition/subtraction of nodes/links. A leading example is the *preferential attachment* model by Barabási and Albert [7], which provides a minimal account of sufficient mechanisms for the emergence of *scale-free* networks [8]. Preferential attachment networks are characterized by power-law degree distributions, $P(k) \sim k^{-\gamma}$ and moderate clustering levels [9].

Heterogeneous preferential attachment models [10, 11] are an extension of the former

model to networks where elements states may induce specific affinities in their interaction. The introduction of heterogeneity preserves the desired scaling of the degree distribution, however it introduces a multiscaling in the degree densities that translates into a richer behaviour of the metrics. In this communication we show that the introduction of heterogeneity also yields a richer scaling in the average network clustering, enabling a better agreement of the extended model with empirical observations.

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