



## **Climate spectrum estimation, aliasing and timescale errors: Algorithms and application to stalagmite and ice core records**

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A power spectrum of a climate process is a powerful tool because it allows to separate short-term from long-term variations and to distinguish between cyclical forcing mechanisms of the climate system and broad-band resonances. This means that spectral analysis permits to learn about the physics of the sampled climate system. However, when having instead of a perfect knowledge only a handful of data contaminated with measurement and proxy errors, the task is to *estimate*, namely the spectrum. This paper gives a short overview of spectrum estimation methods for noisy, perhaps unevenly sampled climate time series. Two extra points are treated in more depth: aliasing and the influence of timescale errors.

*Estimation.* The periodogram is not a suitable estimator of climate spectra because these are not pure harmonic but rather a combination of harmonic signals and a continuous background (i.e., mixed), and the periodogram has 100% (200% at the frequency borders) estimation error for such data. Instead, spectral smoothing has to be applied. The optimal smoothing technique, in a least-squares sense, is Thomson's (1982) multi-taper method. The apparently only drawback of this method today is the unavailability of an implementation for series unevenly sampled in time. Such records are today best analysed using the Lomb–Scargle periodogram combined with Welch's Overlapped

Segment Averaging procedure. An implementation (REDFIT) of this method together with bias correction and a hypothesis test of the AR(1) red-noise alternative was presented by Schulz and Mudelsee (2002).

*Aliasing* occurs when a process with a high-frequency ( $f'$ ) component has been sampled at insufficient temporal resolution. This is the case when (even spacing)

$f' > f_{Ny} = (2d)^{-1}$ , where  $d$  is the time series spacing. Then the power associated with the high frequency is “folded” back into the analysis interval  $[0; f_{Ny}]$ , which produces spurious spectral peaks (“aliases”). One line of how to assess aliasing in an estimated spectrum is to consider the climate physics of the sampled system: time resolution or degree of preservation of high-frequency variations (determined by, e.g., the amount of material consumed for taking a measurement). The other, new line presented here (REDFITmc2 algorithm, mode 'c') is to perform statistical simulations (AR(1) background plus high-frequency sinusoid component) and study the spectrum averaged over the simulations.

*Timescale errors* occur in nearly all analyses of paleoclimate proxy time series. Previous studies of their influence on spectrum estimation made simplifying assumptions and have, up to now, been of limited relevance for the praxis of climate time series analysis. Thomson and Robinson (1996) studied “jittered” timescale errors, where  $d$  is a constant plus the realization of a Gaussian random component. The effect of independent jitter on the spectrum is additional white noise. The jitter model might be less applicable to ice cores, where compaction occurs and the timescale is derived by modeling. Also for other archives, for example stalagmites, more advanced timescale models (heteroscedastic, additive noise) may be better applicable. The problem thereby, however, is that a theoretical analysis of the influence on spectrum estimates approaches intractability. Also in this situation, the simulation algorithm REDFITmc2 (modes 'a' and 'b') can be used to quantify effects of arbitrary types of timescale errors on spectrum estimates. REDFITmc2 outputs (1) the red-noise percentiles (which are higher than in the absence of timescale errors) and (2) the frequency uncertainty of a spectral peak (which can be compared against the analysis bandwidth).

Applications of REDFITmc2 are presented for two exemplary records of paleoclimate: (1) the oxygen isotope record from stalagmite Q5 (Fleitmann et al. 2003), a proxy for Holocene variations of Indian monsoonal rainfall and (2) the deuterium record from the EPICA ice core (Jouzel et al. 2007), a proxy for late Pleistocene temperature changes over Antarctica.

References:

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