



An analysis of three instantaneous unit hydrographs based on the Beta density function

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In this article, three instantaneous unit hydrographs (IUH), based on a Beta density function are described, compared and analysed, with the aim of determining the response of a catchment area by means of characteristic times. The Beta density function is usually written as:

$$B(p, q, x) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} x^{p-1} (1-x)^{q-1}; 0 \leq x \leq 1$$

where p and q are parameters. In the above equation the change of variable x for t has been proposed, in order for the new variable to be comprised between zero and the infinite.

$$x = \frac{t}{t_0 + t}$$

where t_0 represents a constant time. In making the change of variable the Beta function is written as:

$$B(p, q, t_0, t) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} t_0^q \frac{t^{p-1}}{(t_0 + t)^{p+q}}; 0 \leq t < \infty$$

This function depends on three parameters, p , q and t_0 . By giving different values to the last parameter, an IUH depending on two parameters was obtained. Three functions B_0 , $B_{t_{lag}}$ and B_∞ associated with the values of $t_0 \rightarrow 0$; $t_0 = t_{lag}$ and $t_0 \rightarrow \infty$, respectively, have been described. The two parameters of these three functions are expressed by t_{lag} (the difference between the centroids of the direct runoff hydrograph

and the effective precipitation hyetograph) and the relationship between the time to peak, t_p , and t_{lag} (t_p/t_{lag} which represents a form factor of the hydrograph).

These three IUHs have been applied in two basins in the Gipuzkoa province (northern Spain), Aixola of 4.69 km² in 20 events, and Oiartzun of 56.07 km² in 12 events. Using the Powell method for the quadratic minimum error with respect to the observed event, the values of t_{lag} and t_p/t_{lag} were optimized for each of the events and hydrographs proposed.

The Nash and Sutcliffe efficiency index values obtained in the simulations of the three hydrographs were very high in the events in both catchments. The mean of the efficiency values in the basin of Aixola was 0.945, and it was 0.961 in Oiartzun. There were no significant differences between the efficiencies reached by the three IUHs.

The t_{lag} was seen to be the most sensitive parameter compared to the t_p/t_{lag} relationship. It was observed that the mean of the t_{lag} obtained for the B_0 hydrograph was higher than in the $B_{t_{lag}}$ function and the latter, in turn was higher than in the B_∞ one. On the contrary, the mean of the t_p/t_{lag} relationship exhibited an opposite trend to the previous one.

Logically, the mean t_{lag} of the Aixola basin was lower than that of the Oiartzun basin as the latter had a larger surface. It was also observed that the t_p/t_{lag} relationship was greater in Aixola than in Oiartzun, with the latter showing a more advanced t_p compared to the t_{lag} than in Aixola.