



Analytical power series solution to the one-dimensional advection-dispersion equation with asymptotic distance-dependent dispersivity

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The solute transport equation, called advection-dispersion equation is often used to describe the movement of the solute in the groundwater system. Analytical solutions to one-dimensional advection-dispersion equation with a constant dispersion coefficient have been derived under various types of initial and boundary condition.

As it was frequently pointed out in the literatures, dispersivity increases with increasing solute transport distance and scale of the experiments due to macroscopic spatial variation in pore water velocity in geological porous media. Analytically solving the advection–dispersion equation with distance-dependent dispersivity is extremely difficult because that the governing equation coefficients is dependent on the distance variable. This study presents an analytical technique to solve the two-dimensional advection–dispersion equation with asymptotic distance-dependent longitudinal dispersivity for describing solute transport in a uniform flow field. The analytical approach is developed by applying the extended power series method coupled with the Laplace transform. The developed analytical solution is compared with the corresponding numerical solution to asses its accuracy and robustness. Results demonstrate that the breakthrough curves at different spatial locations obtained from the power series solution shows good agreements with those obtained from the numerical solution. However, owing to the limitation of numerical operation for extremely

large values of the power series functions, the developed analytical solution can not be numerically evaluated at the early time when the asymptotic dispersivity is small and/or the characteristic length is large. Moreover, breakthrough curves obtained from the distance-dependent solution are compared with those from the constant dispersivity solution to investigate the relationship of the transport parameters between two models proposed by Mishra and Parker (1990). Our numerical experiments demonstrate that the relationship proposed by Mishra and Parker (1990) is invalid under conditions of small asymptotic dispersivity and/or large characteristic length. The analytical power series solution derived in this study is efficient and can be a useful tool for testing and benchmarking numerical code.