



Perspective on theories of anomalous transport in heterogeneous media

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Subsurface fluid flow and solute transport take place in a multiscale heterogeneous environment. Neither these phenomena nor their host environment can be observed or described with certainty at all scales and locations of relevance. The resulting ambiguity allows alternative conceptualizations of flow and transport and multiple ways of addressing their scale and space-time dependencies. We consider four conceptualizations and representations of passive tracer transport in randomly heterogeneous media: a space-time nonlocal representation which assumes advective-dispersive behavior at some reference support scale and allows velocity to be space-time nonstationary due to forcings and conditioning (which we designate stnADE); a space-time nonlocal representation considering Lagrangian particle motions through a stationary velocity field (stnL); space-fractional representations of advection and dispersion with constant or spatially varying velocity and dispersion coefficients that are nonlocal in space but typically local in time (fADE); and a time-nonlocal representation based on continuous time random walk (CTRW). We describe briefly each of them and offer a perspective on their differences, commonalities, and relative merits as analytical and predictive tools. fADE is shown to be a limited equivalent of stnADE and/or stnL. To render fADE equivalent to stnADE it would be necessary to have zero initial concentration, velocity having stationary fluctuations about a time- and (depending on the form) space-independent mean, constant dispersion coefficient, and cross-fractional space-time derivatives rather than just space-fractional derivatives. CTRW is limited

by a tacit assumption that particle transition rates lack statistical interdependence. A demonstrated one-to-one functional correspondence between CTRW, stnADE and stnL in the case of stationary velocity implies that it is this tacit disregard of spatial dependencies which eliminates space-nonlocality from CTRW, renders it nonlocal only in time, and makes possible solving the underlying generalized master equation in a computationally efficient manner by a Lagrangian continuous time random walk process. Published “observations” of anomalous transport on experimental laboratory or field scales provide no information about the “true” nature of stochastic transport processes on much small scales. In particular, there is no valid theoretical or experimental basis for the assertion [e.g., Berkowitz et al., 2006] that nonlocal transport representations based on assumed stochastic advective-dispersive behavior on some small support scale contradict experimental evidence about the anomalous nature of transport. As stnL and CTRW are limited to stationary fields of particle motion, they are not amenable to conditioning on measured values of quantities that control these motions. That geologic media tend to be structured on a multiplicity of scales additionally imparts a corresponding degree of coherence to such motions which CTRW presently fails to capture. Solutions of stochastic ADE (or its extensions) are unique among the four approaches to anomalous transport we discuss in providing not only predictions of solute concentrations and mass fluxes but also measures of associated prediction errors.