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High-order discontinuous Galerkin methods for solving conservation laws on curved manifolds

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We present in this work a general method for solving partial differential equations on curved manifolds. We apply this technique to the particular case of the sphere with the aim to solve the shallow water equations on the Earth surface. In the framework of a new ocean model based on unstructured grids and finite elements, the discontinuous Galerkin method seems to be a good candidate since it allows the simple use of efficient techniques as high order polynomial function space, adaptivity and error estimation, efficient parallel computing, and exhibits superconvergence properties for the dissipation and dispersion errors.

Classical high order methods for solving the shallow water equations on the sphere consider a three-component velocity $\mathbf{v} = (u, v, w)$, each component being discretized as a scalar field, for a three-dimensional momentum equation. Those techniques do not garantee the velocity vectors to remain tangent to the manifold, and require then the use of explicit time schemes with a projection of the residual and the solution on the local tangent plan at every time step. We propose here a high order discontinuous Galerkin method considering vectorial test functions, taking into account the manifold curvature directly into the discrete operators. The manifold discretization is based on curved triangles with high order polynomial mappings. An implicit DIRK time scheme may then be applied since the discretization itself ensure tangent velocities. The method is validated by comparing the high-order discontinuous Galerkin results to classical benchmarks as the Williamson test cases used in atmospheric computations.