



Response theory for non-equilibrium steady states: causality and generalized Kramers-Kronig relations

V. Lucarini (1,2)

(1) Department of Physics, University of Bologna, Bologna (BO), Italy (2) CINFAI, Camerino (MC), Italy [lucarini@adgb.df.unibo.it]

We consider the general response theory recently proposed by Ruelle for describing the impact of small perturbations to the non-equilibrium steady states resulting from Axiom A dynamical systems. We show that the causality of the response functions entails the possibility of writing a set of Kramers-Kronig (K-K) relations for the corresponding susceptibilities at all orders of nonlinearity. Nonetheless, only a special class of directly observable susceptibilities obey K-K relations. Specific results are provided for the case of arbitrary order harmonic response, which allows for a very comprehensive K-K analysis and the establishment of sum rules connecting the asymptotic behavior of the harmonic generation susceptibility to the short-time response of the perturbed system. These results set in a more general theoretical framework previous findings obtained for optical systems and simple mechanical models, and shed light on the very general impact of considering the principle of causality for testing self-consistency: the described dispersion relations constitute unavoidable benchmarks that any experimental and model generated dataset must obey. These results, taking into account the chaotic hypothesis by Gallavotti and Cohen, might be relevant for climate research. In particular, whereas the fluctuation-dissipation theorem does not work for non-equilibrium systems, because of the non-equivalence between internal and external fluctuations, K-K relations might be robust tools for the definition of a self-consistent theory of climate change.