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Elementary spherical harmonic functions as an aid to solving coupled sliding, interface diffusion and incompressible straining flow around a sphere

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A set of 'elementary' spherical solid harmonic functions with dimensionless multipliers is given for an infinite, slow-moving, incompressible and uncompactable Newtonian viscous medium undergoing rotational straining flow around a spherical inclusion. Their purpose is to make solutions for coupling with general boundary conditions at the spherical interface more readily attainable than they have been hitherto. These conditions include independent linear sliding and diffusion-accommodated normal velocity-difference across the interface. Such conditions are usually not dealt with in classical fluid-dynamics theory but are thought to occur frequently around small objects in naturally deformed rocks.

The pure straining part, $\mathbf{E}^{\infty}$, of the far-field velocity-gradient is included in the spherical surface harmonic part, $S_2(\mathbf{n}) = {}^1\!/_2\, \mathbf{n} \cdot \mathbf{E}^{\infty} \cdot \mathbf{n}$, of the elementary functions, where $\mathbf{n}$ is unit-vector angular position on a sphere. The normal and tangential mechanical properties of the two media are given by velocity/traction ratios or 'moveabilities' of their boundaries. The diffusional 'dilatability' and 'slidability' properties of the interface are defined in a similar way. Coupling of these properties determines the required multipliers for the harmonic functions and, from them, the velocities and velocity-differences at the interface.

Results are given for an impermeable rigid inclusion, a linear interface slidability F_s, a diffusion-limited normal-stress-driven interface-diffusion dilatability F_b, and an impermeable exterior medium whose undisturbed boundary moveability is $F_\mu = a/2\mu$

(μ being viscosity of the medium and a the radius of the inclusion). Normalized dilation (α) and sliding (γ) velocity-difference factors for the interface motion for this case are given by

$\alpha = 5R_b(4R_s + 1) / [6R_b(2R_s + 1) + 5R_s + 2]$

$\gamma = 5R_s(4R_b + 1) / [6R_b(2R_s + 1) + 5R_s + 2]$,

where $R_b \equiv F_b/F_\mu = 12\mu\ w_bML_bV\rho^{-1}a^{-3}$ (a dimensionless dilatability/moveability number) and $R_s \equiv F_s/F_\mu = 2\mu\ F_sa^{-1}$ (a dimensionless slidability/moveability number). w_bML_b, V and ρ are an effective interface-diffusion phenomenological coefficient, molar volume and mass-density of the transported substance. Solutions for α and γ exist in a quadrilateral domain that ranges from 0 (the conventional fully attached condition) to 5/3 times undisturbed velocity (perfect sliding and dilation). α and γ depend generally non-linearly on R_b and R_s, due to the special nature of the medium moveability. They are insensitive to large values of R_b and R_s but depend linearly on them when they are small. For their part, R_b and R_s vary in direct proportion to the interface coefficients and medium viscosity and inversely as a^{-3} and a^{-1} with inclusion size.

α and γ thus tend to be high when the coefficients and/or viscosity are high, and relatively ‘small’ inclusions favour dilation with high α whereas ‘large’ ones favour sliding at low α.