Geophysical Research Abstracts, Vol. 9, 11390, 2007
SRef-ID: 1607-7962/gra/EGU2007-A-11390

Bounds on dissipation in a stress driven flow in a rotating frame

W. Tang (1), C. P. Caulfield (2) and **W. R. Young** (3)

(1) MAE, UC San Diego, (2) BP Institute and DAMTP, University of Cambridge, (3) Scripps Institution of Oceanography, UC San Diego (c.p.caulfield@bpi.cam.ac.uk

We calculate the optimal upper bound and a rigorous lower bound, subject to the assumption of streamwise invariance, on the long-time-averaged mechanical energy dissipation rate $\mathcal{E}$ within the flow of an incompressible viscous fluid of constant kinematic viscosity ν, depth h and rotational rate f, driven by a constant surface stress $\tau = \rho u_\star^2$, where $u_\star$ is the friction velocity. We show that $\mathcal{E} \leq \mathcal{E}_{\max} = \mathcal{E}_{Stokes}$, i.e. the dissipation is bounded above by the dissipation associated with the Stokes flow.

Using an approach similar to the variational "background method" (due to Constantin, Doering and Hopf), we also generate a rigorous lower bound on the energy dissipation rate, subject to the constraints of total power balance and streamwise momentum balance, in the inviscid limit for fixed Tang numbers $T^2 = GE^2$, where $G = \tau h^2/(\rho\nu^2), E = \nu/fh^2$, are the Grashof number and Ekman number. Under the assumption of streamwise invariance as $G \to \infty$ or equivalently $E \to 0$, and by restricting our attention to a particular, analytically tractable, class of background shear profiles analogous to the one used in chapter **3** of Tang, Caulfield & Young (2004) (henceforth TCY04), as the Lagrangean multiplier imposing mean streamwise momentum balance, we show that $\mathcal{E} \geq \mathcal{E}_{\min} = 2\sqrt{3}u_\star^3/h$, a bound that is independent of the flow viscosity.