



## A numerical higher order glacier flow model with Coulomb friction

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A variety of field and theoretical evidence indicates that friction at the base of a glacier can often be described by a Coulomb friction law or ‘regularized’ variants thereof. If  $\tau_b$  is basal shear stress and  $\mathbf{u}_b$  basal velocity, then a Coulomb friction law states that when sliding velocity does not vanish, basal shear stress attains the yield stress,

$$|\tau_b| = \tau_c |\mathbf{u}| \quad \text{when } |\mathbf{u}| > 0.$$

Conversely, if sliding velocity vanishes, basal shear stress is at or below the yield stress

$$|\tau_b| \leq \tau_c \quad \text{when } \mathbf{u} = \mathbf{0}.$$

A regularized form of the Coulomb friction law dispenses with this either-or statement and simply puts

$$\tau_b = \tau_c f(|\mathbf{u}|) \mathbf{u} / |\mathbf{u}|,$$

where  $f$  is a continuous, increasing function with  $f(0) = 0$  and  $f(u) \rightarrow 1$  as  $u \rightarrow \infty$ .

As discussed in Schoof (2004 & 2006), the either-or nature of a Coulomb friction law engenders a free boundary at the bed: this boundary is the line separating regions where the bed fails plastically from those where it does not. In practical terms, this free boundary can potentially be identified with the location with ice stream margins. In modelling ice flow with a Coulomb friction law, depth-integrated models such as the shallow ice and shelfy stream approximations are problematic because the natural velocity boundary condition at the free boundary between regions of slip and no slip is one of zero velocity. This would suggest zero inflow of ice across the shear margins of an ice stream, which is clearly not a practical proposition.

A suitable alternative is the use of a higher-order ice flow model (Blatter, 1995), which allows shearing in the vertical as well as lateral and longitudinal stresses to be re-

solved, albeit at greater numerical expense. Here we demonstrate how Blatter's model with Coulomb-type basal boundary conditions can be solved effectively using finite elements, and how this method automatically traces the location of the slip/no slip free boundary at the bed.