



A differential delay model of ENSO variability: quantitative predictability and structural instability

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Climate models are among the most detailed and sophisticated models of natural phenomena in existence. Still, the lack of robust and efficient parametrizations for general circulation models (GCMs), along with the inherent sensitivity to initial data and the complex nonlinearities involved, present a major and persistent obstacle to narrowing the range of estimates for end-of-century warming. Claims about future changes in the distribution of climatic extrema are thus particularly questionable. Continuing efforts at brute-force tuning the large number of model parameters present in GCMs do not appear to achieve the desired reduction of uncertainties.

Andronov and Pontryagin proposed 70 years ago an interesting way to consider the question of model robustness, namely structural stability, in which the qualitative, topological behavior of solutions was used as a criterion. Unfortunately, many “real-world” systems — physical, chemical and biological — proved to be structurally unstable.

As an illustration of these concepts, we investigate here a very simple model, driven by the seasonal cycle, for studies of the El Niño-Southern Oscillation (ENSO). This model is governed by a differential delay equation (DDE) with a single delay and periodic forcing. Like many of its more or less detailed and realistic precursors, this model exhibits a quasi-periodic transition to chaotic behavior, often called a Devil’s staircase. We focus on a detailed analysis of the structural stability and dynamical properties of the model over a much broader parameter range than previously done.

This simple ENSO model is characterized by structural instability in a broad range

of parameters. We describe the mechanisms of the observed instability, connect our findings to ENSO phenomenology, and address questions of quantitative changes in model statistics as the topology of the model attractor changes.