The $L^{1/2}$ particle number law in rain

S. Lovejoy (1), D. Schertzer (2)

(1) Physics dept., McGill U., 3600 University st., Montréal, Canada
(lovejoy@physics.mcgill.ca (2) CEREVE, Ecole Nationale des Ponts et Chaussées, 6-8,
avenue Blaise Pascal, Cité Descartes, 77455 MARNE-LA-VALLEE Cedex, (1) Physics dept.,
McGill U., 3600 University st., Montréal, Canada
(lovejoy@physics.mcgill.ca/1-514-398-6537)

The stereophotography of rain drops 10m^3 volumes (the "HYDROP" experiment
.................[*Desaulniers-Soucy., et al.*, 2001]) has for the first time made it possible
to empirically determine both the scaling properties and small scale scaling limits
of both the liquid water density (ρ) and particle number density fields (n). In previous
work, these data established that –due to the coupling of the drops and turbulence, that
for scales larger than 30-50cm, the large scale density fluctuations were multifractal.
In this paper, we show that the decoupling scale is (as expected) the relaxation scale
for the drops and that at larger scales, the spectra are indeed close to those predicted
for passive scalars by Corrsin-Obukhov theory (corresponding to density fluctuations
$\Delta\rho \approx l^{1/3}$ i.e. to a classical $k^{-5/3}$ spectrum for ρ) which assumes that the passive
scalar variance of liquid water density is conserved from scale to scale. This is the
first direct link between precipitation and passive scalar turbulence theory of which
we are aware. However, the key link between turbulence and precipitation is via the
particle number density (n). We show that since drop coalescence processes conserve
particle mass but not number densities, that the number density flux is also a turbu-
lent quadratic invariant but with a transfer time scale which is different than for the
liquid water variance flux. From dimensional analysis, we theoretically predict a new
$\Delta n \approx l^{1/2}$ for fluctuations in n over a distance l corresponding to a k^{-2} spectrum for n
which we confirm is close to observations.

Finally, we show how to model these number density and liquid density statistics us-
ing coupled multifractal cascade processes. These in turn can control a multifractal
compound Poisson process which models the positions and volumes of individual
rain drops. Numerical simulations spanning the range 1cm to 1000km can be readily

produced. These simulations can be used for simulating radar reflectivity factors, effective radar reflectivity factors; extensions of the model can be used to simulate rain rates and rain gauges. These models can thus potentially solve various precipitation observer problems.