



Modelling advective transport with the exact-analytical solution of two-dimensional unconfined groundwater flow

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The appropriate mathematical representation of unconfined groundwater flow is an important problem in groundwater hydrology. The by far most common approach in an analytical treatment of this problem is solving the linearized Boussinesq-Equation assuming the validity of the Dupuit assumption of essential horizontal flow. This approach fails when strong vertical hydraulic gradients at the groundwater surface or within the groundwater body occur. This is often encountered in the vicinity of local groundwater flow phenomena like e.g. the interaction between a canal and an adjacent aquifer, groundwater mounding due to artificial recharge or irrigation losses or when radial flow in the vicinity of drains or horizontal wells occurs. In many cases the simulation of such systems aims on water quality issues where the groundwater flow velocities derived from a flow model are used to solve the transport equations.

An effective method for particle-tracking analysis of groundwater flow paths and travel times in unconfined groundwater flow systems with significant vertical hydraulic gradients is presented. It applies to the groundwater flow in a cross-section of a homogeneous aquifer where the groundwater flow is described by the Laplace-Equation subject to the nonlinear free surface boundary condition which includes recharge and/or drainage effecting the groundwater surface. Edenhofer [2000] and Schmitz [2007] presented the exact-analytical solution for this problem under steady-state conditions including additionally inner sources or sinks of groundwater. It is an appropriate approach to describe local groundwater flow where boundary conditions induce flow patterns with a significant vertical hydraulic gradient and where the

Dupuit assumptions are no longer valid. Extending this solution an analytical expression for the evaluation of the pressure head distribution and its derivatives at any point within the groundwater body is derived. Thus contaminant particles can be tracked without the need for interpolation of flow velocities between e.g. cell nodes. Groundwater flow paths and travel/transit times are computed in a fast post-processing operation. Sample applications include the computation of groundwater transit times from a groundwater mound to a fixed-head boundary and the analysis of groundwater flow paths and travel times in a drained aquifer under an irrigated field.

Edenhofer, J. & G.H. Schmitz (2001): Pressure distribution in a Semi-Infinite Horizontal Aquifer with Steep Gradients Due to Steady Recharge and/or Drainage: The Exact Explicit Solution, *Transp. in Porous Media*, 45, 347-366.

Schmitz G.H., Edenhofer, J., Weber, J.G., Fröhner, A., Müller, A. (2007) : Analytical solution of the nonlinearized 2D-groundwater flow problem in shallow aquifers with steep hydraulic gradients. Submitted to *Journal of Hydrology*.