



Rigorous treatment of limit cycles in periodically forced quasi-geostrophic models

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Quasi-geostrophic (QG) models are still extensively used in the study of low-frequency variability in the atmosphere and oceans. A number of mathematically rigorous results have been obtained so far for the autonomous (i.e., steady-forcing) QG equations in a periodic-channel geometry. In both meteorological and oceanographic settings, though, the seasonal forcing plays a major role in the low-frequency variability, from subseasonal to interannual and interdecadal. Our current work deals, therefore, with periodically forced QG systems, in a closed rectangular basin.

The response of a QG system in such a basin to periodic forcing is expected to be itself periodic, provided the flow regime is close to the linear Sverdrup regime. Standard existence theorems, however, provide no information on either the amplitude of the resulting periodic solution or on the localization of the associated limit cycle in phase space. We propose here a novel theoretical approach that uses perturbative methods for answering these two questions. We show that the periodically forced QG equations inherit certain properties from the autonomous QG equations that allow us to localize stable, as well as unstable limit cycles in phase space, even for highly nonlinear regimes, in which inertial recirculations play a major role. Moreover, fine estimates of the limit cycle amplitudes can be obtained as bifurcation parameters (such as the Reynolds number or the mean-annual wind-stress forcing) are varied.

Our approach can be applied to a large class of non-autonomous partial differential equations governing geophysical flows. We will then discuss extensions of these results to more complicated time-dependent forcing.