



Orographically-generated nonlinear waves and shocks

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Since early manned space flight orographically forced cloud patterns have been described in terms of the single isolated shock structure of shallow water flow or, equivalently, compressible fluid flow. Stevenson (1969) describes a NASA photograph of Guadalupe Island, Baja California taken during the Gemini-V flight where a low layer of stratocumulus cloud moves at 6-10 knots past the island whose peaks reach 4,500 feet and thereby project through, and interfere with, the cloud layer. Stevenson notes that a “shock or bow” wave spreads from the north end of the island, “similar to waves formed by a ship moving through water”. This talk first briefly reviews these observations, and then reports experimental observations of finite amplitude interfacial waves forced by a surface-mounted obstacle towed through a two-layer fluid. The observations are modelled by a simple forced Benjamin-Davis-Acrivos equation and comparison between integrations of both linear and nonlinear problems shows the fundamental nonlinearity of the near-critical flow patterns. Finally, theory is presented for non-dispersive and weakly dispersive free surface flows over axisymmetric obstacles of nondimensional height M , measured relative to the layer depth. For transcritical flow, where the Froude number F of the oncoming flow is close to unity, a similarity theory is developed for small M , and for non-dispersive flow the problem is shown to be isomorphic to that of transonic flow of a compressible gas over a thin aerofoil. The nondimensional drag exerted by the obstacle on the flow takes the form $D(\Gamma)M^{5/3}$, where $\Gamma = (F - 1)M^{-2/3}$ is a transcritical similarity parameter, and D is a function that depends on the shape of the ‘equivalent aerofoil’ specific to the obstacle. The theory is verified numerically by comparing results from a shock-capturing shallow water model with corresponding solutions of the transonic small disturbance equation, and is generally found to be accurate for $M < 0.4$ and $|\Gamma| < 1$. In weakly dispersive flow the equivalent aerofoil becomes the boundary condition for the Kadomtsev-Petviashvili equation and (multiple) solitary waves replace hydraulic jumps in the flows.