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Upper bounds for the long-time averaged buoyancy flux in plane stratified Couette flow subject to a mixing efficiency constraint

W. Tang (1), R. R. Kerswell (2), C. P. Caulfield (3)

(1) MAE, UC San Diego, (2) Mathematics, University of Bristol, (3) BP Institute and DAMTP, University of Cambridge

We derive non-trivial upper bounds for the long-time averaged vertical buoyancy flux $\mathcal{B}^* := \langle \rho u_3 \rangle g/\rho_0$ (where u_3 is the vertical velocity, g is the acceleration due to gravity, and angled brackets denote volume and time averaging) for stably stratified Couette flow: i.e. the flow of a Boussinesq fluid (with reference density ρ_0, kinematic viscosity ν, and thermal diffusivity κ) confined between two parallel horizontal plates separated by a distance d, which are driven at a constant relative velocity ΔU, and are maintained at a constant (statically stable) temperature difference leading to a constant density difference $\Delta\rho$. We construct the bound by means of a numerical solution to the "background method" variation problem as formulated by Constantin and Doering using a one dimensional, uni-directional background. We require that the mean flow is streamwise independent and statistically steady. Furthermore, we impose the plausible coupling constraint that a fixed fraction of the energy input into the system by the driving plates (above that required to maintain a purely laminar flow) leads to enhanced irreversible mixing within the flow, i.e. we require a coupling such that that $\mathcal{B}^* = \Gamma_c(\mathcal{E}^* - \mathcal{E}^*)$, where $\mathcal{E}^*$ is the total mechanical energy dissipation rate, and $\mathcal{E}^*$ is the total mechanical energy dissipation rate associated with a purely parallel, laminar flow. We calculate this bound up to asymptotically large Reynolds numbers for a range of choices of coupling parameters Γ_c and bulk Richardson numbers $J = g\Delta\rho d/(\rho_0 \Delta U^2)$ of the flow. For all values of Re, we find that the calculated upper bound increases with J. However, there is always a maximum possible value of $J_{\max}(Re, \Gamma_c)$, at which it becomes impossible to impose the new constraint, and the density field and velocity field become decoupled. $J_{\max}$ increases with Re, but is a non-monotonic function of Γ_c, with for a given Re, a maximum apparently at

$\Gamma_c = 1$. The value of the bound on the buoyancy flux at $J_{\max}$ is also a non-monotonic function of Γ_c, with $\Gamma_c = 1/2$ leading to the largest possible values as $Re \to \infty$, consistently with the upper bound presented previously in Caulfield, Tang & Plasting (2004) where this coupling constraint was not imposed. The asymptotic scaling of the new coupled bound is the same as the previously calculated value, with dimensionally $\mathcal{B}^*_{\max} = \mathcal{O}(U^3/d)$, independent of the flow viscosity. At any particular value of Re, the previously calculated bounding solution may be associated with a specific value of Γ_c. Imposing the coupling constraint with that value of Γ_c, as $J \to J_{\max}$, the new bound approaches from below the previously calculated bound exactly.