



When the Hasselmann equation fails: "Fast" nonlinear evolution of water wave spectra

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The cornerstone of the established view on wind wave nonlinear evolution is that due to nonlinear quartet interactions the spectra of a random water wave field evolve on the ε^{-4} timescale, where ε is the characteristic wave steepness. We demonstrate by direct numerical simulations (DNS) that when the wave field is far from the vicinity of an equilibrium state, spectra evolve much faster, on the ε^{-2} timescale. The classical kinetic (or Hasselmann) equation is found to describe only a certain vicinity of an equilibrium state. The departure from the equilibrium can be caused by a variety of factors (rapid change of wind, presence of swell, presence of inhomogeneous currents or internal waves, interactions with bodies and shores, etc).

The specific example is concerned with wind wave-swell interaction. Such situations are typical of storms where locally generated wind waves are superposed upon remotely generated swell, with lower frequency and different direction. The simulations were carried out for initially bi-modal spectra, consisting of a wind-wave spectrum of the standard JONSWAP form and low-frequency swell coming from different angles. We employ a particular DNS approach based on the integrodifferential Zakharov equation for simulation of long term evolution of random water wave fields. The numerical method includes building of a non-regular grid of $O(10^3 - 10^4)$ harmonics coupled by $O(10^7)$ approximately resonant interactions. We show that for realistic wave amplitudes, the initially bi-modal spectrum quickly becomes unimodal, with a low-frequency swell peak, closely resembling the spectra obtained with uni-directional wind conditions. At the same time, directional spectra are shown to be skewed, corresponding to observations of hurricane directional spectra by Young (2006). We demonstrate that the timescale of adjustment of the spectra is $O(\varepsilon^{-2})$, i.e. much faster than the timescale predicted by the classical kinetic equation. This fast evolution has a threshold character: for sufficiently low amplitudes of both swell and wind waves or

sufficiently wide separation of the peaks in the wavevector space, their interaction is weak, and the wave field can be considered as being close to equilibrium. A modified kinetic equation able to describe adequately both the $O(\varepsilon^{-2})$ fast and $O(\varepsilon^{-4})$ slow evolution of wave spectra has been also derived.

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