



Stability and transitions of hetonic quartets and baroclinic modons

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A new concept, a hetonic quartet, is presented. A hetonic quartet is a two-layer ensemble of four synchronously translating quasigeostrophic discrete vortices aligned perpendicularly to the axis of their translation. On the f -plane, a hetonic quartet is made up of conventional point vortices (confined to either the upper or the lower layer) with specially fitted circulations and distances between each other, the upper- and lower-layer vortices being opposite in sign and located symmetrically about the translation axis. On the beta-plane, a two-layer version of the modulated point vortex model is employed. A hetonic quartet can be pictured as a pair of aligned synchronously translating hetons. Hetons and hetonic quartets share some traits with baroclinic modons and, therefore, offer a relatively simple, finite-dimensional model for exploring the modon stability and transitions.

Baroclinic modons, i.e., localized steady-state solutions to the nonlinear equations of potential vorticity (PV) conservation in a (differentially) rotating stratified fluid, represent a paradigm for coherent structures in geophysical flows. A baroclinic modon appears as two PV chunks of opposite signs, which reside at different depths (one in the upper layers and the other in the lower layers) and are shifted relative to each other in the north-south direction. A hetonic quartet represents a discrete model for a two-layer modon whose upper- and lower-layer PV chunks overlap considerably, while a heton models a non-overlapping modon.

Based on the similarity between baroclinic modons, on the one hand, and hetons and hetonic quartets, on the other, a plausible scenario of the transitions observed in baroclinic modons with moderate riders is suggested, with the substantial stability of the ultimate modon state being attributed to the lack of overlap in it. The key questions

addressed are the following. Why do non-overlapping modons remain essentially insensitive to the prolonged action of small perturbations? Why does, in contrast, an overlapping modon subjected to the same perturbations, after a sufficiently long period of steady propagation, suddenly, change its parameters, making a transition to a non-overlapping modon state?

These questions, being closely linked to the modon stability, are answered via the stability analysis of hetonic quartets. A necessary and sufficient condition for the non-linear stability of an f -plane hetonic quartet is established using the analytical methods of Hamiltonian dynamics, while on the β -plane, a linear stability condition is determined. Both hetons and overlapping hetonic quartets are stable. However, whereas a heton is a rigid construction, a stable hetonic quartet, when slightly moved off equilibrium, undergoes elastic oscillations. Because a stable overlapping hetonic quartet is located near the stability border in the parameter space, periodically or continuously acting small perturbations force such a hetonic quartet to split up into two hetons, each traveling at different speeds. This splitting imitates the process of riddance of overlap attended by shedding some PV, observed in modons.