



A phenomenological model for the preferential concentration of heavy particles in turbulent flows

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Understanding the dynamics of small-size tracer particles or of a passive field transported by an incompressible turbulent flow plays an important role in the description of several natural and industrial phenomena. For instance it is well known that turbulence has the property to induce an efficient mixing over the whole range of length and time scales spanned by the turbulent cascade of kinetic energy. Describing quantitatively such a mixing has consequences in the design of engines, in the prediction of pollutant dispersion or in the development of climate models accounting for transport of salinity and temperature by large-scale ocean streams.

However, in some settings, the suspended particles have a finite size and a mass density very different from that of the fluid. Thus they can hardly be modeled by tracers because they have inertia. In order to fully describe the dynamics of such inertial particles, one has to consider many forces that are exerted by the fluid even in the simple approximation where the particle is a hard sphere much smaller than the smallest active scale of the fluid flow. Nevertheless the dynamics drastically simplifies in the asymptotics of particles much heavier than the carrier fluid. In that case, and when buoyancy is neglected, they interact with the flow only through a viscous drag, so that their trajectories are solutions to the Newton equation :

$$\tau \ddot{X} = - \left[\dot{X} - u(X, t) \right], \quad (1)$$

where u denotes the underlying fluid velocity field and τ is the response time of the particles. Even if the carrier fluid is incompressible, the dynamics of such heavy particles lags behind that of the fluid and is not volume-preserving. At large times particles

concentrate onto singular sets evolving with the fluid motion, leading to the appearance of strong spatial inhomogeneities dubbed *preferential concentrations*. At the experimental level such inhomogeneities have been known for a long time (see [1] for a review). At present the statistical description of particle concentration is a largely open question with many applications. We mention the formation of rain droplets in warm clouds, the coexistence of plankton species, the dispersion in the atmosphere of spores, dust, pollen, and chemicals, and the formation of planets by accretion of dust in gaseous circumstellar disks.

The dynamics of inertial particles in turbulent flows involves a competition between two effects: on the one hand particles have a dissipative dynamics, leading to the convergence of their trajectories onto a dynamical attractor [2], and on the other hand, the particles are ejected from the coherent vortical structures of the flow by centrifugal inertial forces [3]. The simultaneous presence of these two mechanisms has so far led to the failure of all attempts made to obtain analytically the dynamical properties or the mass distribution of inertial particles. In order to circumvent such difficulties a simple idea is to tackle independently the two aspects by studying toy models, either for the fluid velocity field, or for the particle dynamics that are hopefully relevant in some asymptotics (small or large response times, large observation scales . . .). Recently an important effort has been made in the understanding of the dynamics of particles in flows that are δ -correlated in time [4, 5]. Such settings trivially rule out the presence of any persistent structure in the flow and any observed concentrations can only stem from dissipative dynamics.

Recent numerical studies in fully developed turbulent flows [6] showed that the spatial distribution of particles at lengthscales within the inertial range are strongly influenced by the presence of voids at all active scales spanned by the turbulent cascade of kinetic energy. The presence of these voids has a noticeable statistical signature on the probability density function of the coarse-grained mass of particles which displays an algebraic tail at small values. To understand at least from a qualitative and phenomenological viewpoint such phenomena, it is clearly important to consider flows with persistent vortical structures which are ejecting heavy particles. For this purpose, we introduce a toy model where the vorticity field of the carrier flow is assumed piecewise constant in both time and space and takes either a finite fixed value ω or vanishes (see [7] for details). In addition to this crude simplification of the spatial structure of the fluid velocity field we assume that the particle mass dynamics obeys the following rule: during each time step there is a mass transfer between the cells having vorticity ω toward the neighboring cells where the vorticity vanishes. The amount of mass escaping to neighbors is at most a fixed fraction γ of the mass initially contained in the ejecting cell. We show that such a simplified dynamics reproduces many aspects of the

mass distribution of heavy particles in incompressible flow. In particular, we show that the probability distribution function of the mass of inertial particles has an algebraic tail at small values and decreases as $\exp(-A m \log m)$ when m is large. Analytical predictions are confirmed by numerical experiments in one and two dimensions.

References

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