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Analysis of the impact of model nonlinearities in solutions to stochastic inverse problems

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To better understand the relationship between the properties of nonlinear models and the solution to stochastic inverse problems in Geoscience applications an explicit analysis of posterior probability density functions (pdf_s) is applied to several relatively simple models with increasing complexity. The effect of the nonlinear model solution on the posterior pdf, in which the model effects a mapping from control parameter space into observation space, is controlled by modeling and observation errors, observation spatial and temporal distribution, and assumptions made about the characteristics of the prior probability density function. The analysis performed in this study, which is based on the general stochastic inverse problem formulated by Mosegaard and Tarantola, addresses each of these aspects. It consists of explicit evaluation of discrete pdf_s which, by conjunction, give rise to the posterior joint pdf. Results of a Markov chain Monte Carlo (MCMC) inversion algorithm are used in the analysis of time evolving models to provide an appropriate reference point for the analysis and also to estimate the prior covariance matrix when it is needed. The primary conclusions derived from the analysis are:

- Nonmonotonic transformations from the control parameter into observation space result in the possibility of a multimodal posterior pdf. Whether multiple modes are, in fact, observed depends on the information content of the observations relative to the control state and on the observation uncertainties.
- The model solution must be a monotonic function with nonzero slope in a region

spanned by observation uncertainty to render an informative unimodal posterior pdf.

- Observations with identical accuracy taken from different observation times do not contribute equally to the final solution. Instead, observation influence on the posterior pdf depends on the nature of nonlinearity in the process model at each time. The contribution of observations from different times is equivalent only for linear models.
- Assumption of relatively large observation errors increases the chances the posterior pdf will exhibit multiple modes for a given nonlinear model. Under this condition the maximum likelihood solution may be preferred central estimate.
- Increasing an assumed-Gaussian model error widens the final posterior pdf, but does not increase the potential for a multimodal solution.
- Under good observational constraint, which is equivalent to relatively small observation error coupled with monotonic model behavior, the posterior solution is well represented either by the maximum likelihood or central mean estimate. The exception is the case in which the model function tends to the exponential, which then leads to the lognormal posterior pdf.
- Assumption of a Gaussian prior pdf has a detrimental effect on the mean of the posterior pdf when errors in the observations and model are relatively large. The negative effect of the Gaussian prior assumption increases with increasing model error.