



Green's functions for the one-dimensional wave equation with variable coefficients

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BACKGROUND

In previous studies [1] [2] Seymour and Varley demonstrated the technique for solving the one-dimensional wave equation for the special case when the sound speed c depends only on x , i.e. $c = c(x)$. They showed that an exact solution could be obtained for an acoustic wave for a wide class of spatial dependencies of the wave speed and that the exact solution could be expressed in terms of the exact solution for the one-dimensional wave equation for a constant sound speed. In particular, they further demonstrated that the expansion coefficients could be obtained as a solution to $2N-1$ coupled Riccati equations, yielding a solution for the wave speed, $c(x)$ in terms of $2N+1$ arbitrary constants, for a large class of spatial dependencies of the sound speed. In this analysis, there was no restriction on the magnitude of the sound speed gradient relative to the wavelength, as is common in asymptotic methods.

We propose to extend the method in [2] to the case where sources exist for the one-dimensional wave equation with a spatially variable sound speed. There is first an equivalent source problem that will be solved for the constant coefficient one-dimensional wave equation. It will be then demonstrated how the solutions obtained for the constant coefficient case can be directly employed in the finite expansions for the source-driven equation with variable sound speed. The formalism of the Riccati technique will be presented as well as results for simple classes of inhomogeneities. Finally, the extension of this finite expansion technique for the acoustic wave equation, in two and three dimensions, will be outlined.

BIBLIOGRAPHY

[1] B. Seymour and E. Varley, "Exact Representation for Acoustical Waves When the Sound Speed Varies in Space and Time," *Studies in Applied Mathematics*, vol. 76, pp. 1-35, 1987.

[2] E. Varley and B. Seymour and , "A Method for Obtaining Exact solutions to Partial Differential Equations with Variable Coefficients," *Studies in Applied Mathematics*, vol. 78, pp. 183-2255, 1988.