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Modal analysis of dispersion and dissipation properties applied to Poincaré, Kelvin and Rossby waves with discontinuous Galerkin finite element method

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A general method is proposed for analyzing dispersion and dissipation properties of any numerical scheme on any kind of structured or unstructured grid. This method is applied to the discontinuous Galerkin discretization of the rotating shallow water equations. It is able to deal with non periodic boundary conditions and with dispersive and non dispersive waves, i.e. waves with non linear dispersion relation and continuous eigenfunctions which are not sine or cosine functions. An eigenvalue analysis of the discrete operator is performed. An error estimator is initially applied to each eigenfunction in order to distinguish resolved and unresolved modes. Some kind of Fourier analysis is applied to the eigenfunctions in order to determine the numerical wave number spectrum. The discrete dispersion relation is then constructed using the wavenumbers (eigenfunctions) and frequencies (eigenvalues). Both spatial and spectral superconvergence properties for all waves are found out in one analysis on any mesh. Our method allows to demonstrate that the DG method is able to predict Poincaré and Kelvin waves with a super-accuracy at polynomial order 2p+3. It is a very useful tool to compare in an accurate way different numerical methods and to investigate some convergence properties of one scheme and thus to help ranking numerical methods on the basis of dissipation and dispersion errors.