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Markov Chain Monte Carlo algorithm for Bayesian reconstruction of a dynamical system by noisy chaotic time series and its application to prognosis of bifurcations

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In terms of constructing solutions, one of the most complicated cases in the problem of reconstructing dynamical systems by time series (TS) is the case of chaotic series corrupted by the measurement noise. Existence of the noise makes it necessary to describe the evolution of the dynamic system (DS) statistically: DS states should be regarded as (vector) random values. In its turn, it leads to a statistical description of the parameters of the DS evolution operator, constructing which is the objective of the reconstruction. A consistent way to make this description can be proposed in the framework of the Bayesian approach, which ensures obtaining the explicit expression for the probability density of random values (parameters of the evolution operator and the DS states), which takes into account both the known probability distribution of the noise, and the a priori information about DS. Specifically, recently it has been shown [1] that the information about the chaotic regime of DS functioning, despite the exponentially fast divergence of phase trajectories on the chaotic attractor, makes it possible to perform Bayesian reconstruction of the system over an infinitely extended TS. This fact provides the possibility to use the Bayesian approach in the situation when sufficiently long TS are a precondition of the reconstruction. An example of this kind is reconstruction of DS, for which the functional form of the evolution operator is unknown.

A key “technical” issues of the Bayesian reconstruction is (numerical) construction of a statistical assembly of random quantities distributed in accordance with the obtained probability density. This problem can be solved using the MCMC (Markov Chain Monte Carlo) algorithms [2]. However, strong dependence of the resource capacity of these algorithms on the dimensionality of the distribution makes them inefficient even for comparatively short TS. This report proposes a method ensuring construction of a representative sampling for a portion of the complete set of arguments of the initial probability distribution. The concept of the method is the approximate (asymptotically accurate) analytical integration of the initial multidimensional distribution over “unnecessary” arguments. For example, when reconstructing DS from noisy chaotic TS, the objective is to construct a statistical assembly of evolution operator parameters, whereas the major part of the probability distribution arguments is comprised by DS states. The efficiency of the proposed method is demonstrated, first, by comparing, for the sake of testing, the rates of convergence of the parameter assembly construction procedure to the “complete” probability distribution and that integrated over "unnecessary" arguments. Second, application of this method solves successfully the problem which cannot be solved using the available MCMC algorithms: bifurcations of unknown DS are correctly forecasted from the noisy non-stationary chaotic long-duration TS.

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[2] Gilks W.R., Richardson S., and Spiegelhalter D.J. Markov Chain Monte Carlo in Practice. London: Chapman and Hall, 1996.