



Instantaneous shock formation in Hertzian media

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Nonlinear compressional waves incident on a granular medium subject to the Hertzian nonlinearity of order $3/2$ in the strain rate exhibit behavior distinct from that which would be observed in a quadratically nonlinear medium. An initially continuous waveform can form a shock instantaneously, whereas for quadratic nonlinearity, shock formation requires a finite time. When a wave has a zero crossing with negative slope in the direction of propagation, the the amplitude of the shock discontinuity in Hertzian media starts at zero and grows with time. Conversely, near a zero crossing with positive slope in the direction of propagation the wave slope is immediately forced to zero. These statements are supported by numerical and analytical solutions. One implication of such behavior is that Hertzian nonlinearity leaves a recognizable signature on a propagating wave. Despite these exotic behaviors, nonlinear plane waves in Hertzian media have been shown to remain stable and evolve toward self similar form $f(x/t)$ at a predictable rate [J. Acoust. Soc. Am. 120, 3503 - 3508, 2006]. Work supported by the U. S. Office of Naval Research.