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Separated estimation of the shift, rotation and scale parameters of the Burša-Wolf transformation

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The procedures of estimation of the seven parameters of the Burša-Wolf transformation, based on either matrix algebra or function minimalization, can be interpreted as a kind of geodetic network adjustment. During these procedures, all the seven parameters are estimated simultaneously. The consequence is, that the resulted parameter set should be handled as a uniform one; not a single parameter has real physical or geometric meaning without interpreting all the others.

The rotation parameters of the transformation can be interpreted as functions of the rotation center and the rotation angle between the the orientation of the two datum systems. We can set the rotation center according to the initial geodetic information of the investigated datum. In most cases, the fundamental point can be used as the center of rotation. Using this *a priori* knowledge, it is possible to define first the shift parameters of the Molodensky transformation, and then searching the rotation angle providing the best results throughout the investigated basepoint set. This is a simple one-dimension minimalization. As the vertical coordinate (the ellipsoid height) is practically independent from the rotations, it is possible to find the scale factor, providing the best vertical fitting average for the investigated points after defining the rotational components, again by a separated one-dimension minimalization.

The first step of the method is to determine the dX, dY and dZ datum shift parameters (the elements of the vector, connecting the centers of the two datum ellipsoids), according to the Molodensky-type parametrization. If we know the coordinates of the fundamental point (or any common point) and the geoid undulation values on both system, the parameters are:

$$\begin{aligned} dX &= (N_1 + n_1)\cos\Phi_1\cos\Lambda_1 - (N_2 + n_2)\cos\Phi_2\cos\Lambda_2 \\ dY &= (N_1 + n_1)\cos\Phi_1\sin\Lambda_1 - (N_2 + n_2)\cos\Phi_2\sin\Lambda_2 \\ dZ &= \left[N_1\left(1 - e_2^2\right) + n_1\right]\sin\Phi_1 - \left[N_2\left(1 - e_2^2\right) + n_2\right]\sin\Phi_2 \end{aligned} \tag{1}$$

where N is the curvature in the prime vertical, e is the eccenticity and n is the geoid undulation, the indices indicate the data of the first and the second datums. Note that in case of local datums, the geoid undulation at the fundamental point is usually set to zero. In most cases, the target datum is the WGS84. Geoid undulation data, compared to the WGS84 can be obtained from global geoid models, eg. the EGM96.

The shift parameters, resulted by the Eq. (1) can be used stand-alone for transformation. If the other four parameters of the Burša-Wolf transformation are set to zero, these work alone as simple Molodensky-type parameters, although it provides significant remanent error throughout an investigated geodetic basepoint system. Now let's estimate the other four parameters to obtain the optimal fit at the basepoint.

In the spheric approximation, the spheric coordinates of the rotation center (ϕ,λ) and the angle of the rotation (α) can be expressed as follows:

$$\varphi = \arctan\left(\frac{r_Z}{\sqrt{r_X^2 + r_Y^2}}\right) \tag{2}$$

$$\lambda = \arctan\frac{r_Y}{r_X} \tag{3}$$

$$\alpha = \sqrt{r_X^2 + r_Y^2 + r_Z^2} \tag{4}$$

where ϕ and λ are the spheric coordinates of the rotation center, and α is the angle of the rotation around the center.

Inverting the formulae (2)-(4), the rotational components can be expressed in the well known form as, in case of the spheric approximation:

$$r_X = \alpha\cos\Phi\cos\Lambda \tag{5}$$

$$r_Y = \alpha\cos\Phi\sin\Lambda \tag{6}$$

$$r_Z = \alpha\sin\Phi \tag{7}$$

In ellipsoidal case:

$$r_X = \frac{\alpha\cos\Phi\cos\Lambda}{\sqrt{1 - e^2\sin^2\Phi}} \tag{8}$$

$$r_Y = \frac{\alpha \cos \Phi \sin \Lambda}{\sqrt{1 - e^2 \sin^2 \Phi}} \tag{9}$$

$$r_Z = \frac{\left(1 - e^2\right) \alpha \sin \Phi}{\sqrt{1 - e^2 \sin^2 \Phi}} \tag{10}$$

where e is the eccentricity of the starting datum ellipsoid. Now all we have to do is to set ϕ and λ to the coordinates of the fundamental point or the used common basepont and seek an α rotation value, providing the less horizontal error throughout the used basepoint set. This vay, we reduced the estimation of the three rotation parameters to a single-variable minimalization of the remanent errors. It can be easily completed by iteration, even by a manual one. Upon founding the best fitting α rotation value, substitute it back to Eqs. (5)-(7) or Eqs. (8)-(10), to get the three rotation parameters.

If vertical coordinates (the ellipsoid height) should be taken into account, it is worth to use that the hight is practically independent from the rotations. Therefore, it is possible to find the scale factor, providing the best vertical fitting average for the investigated points after defining the rotational components, also by a simple, indepentent one-dimension function minimalization, practically by an iteration.

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