



Breeds of the Reduced Rank Square Root Kalman filter: regression analysis framework as a source of inspiration

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Since appearance the Reduced Rank Square Root Kalman filter (RRSQRT KF) in the mid 90's the scheme became a popular choice as a deterministic suboptimal Kalman filter procedure, applicable for dynamic systems with high dimensionality.

The RRSQRT KF scheme has an intrinsic step - reduction or truncation, directed on preserving constant low rank of the covariance square root approximation. A classical reduction procedure suggesting to work with small S^T*S rather than large $S*S^T$ matrix is time effective, but it was not clear whether it is correct for multiphase systems. Several approaches on tackling the problem were introduced, including renormalization before truncation with different normalization techniques.

Alternatively, other truncation scheme were suggested not utilizing a dimensionality S^T*S trick. To name here Reduced Rank Transform Square Root Kalman filter (Gilljns et al. 2006), Truncation Enhanced RRSQRT Kalman filter (Treebushny, Madsen, 2005).

Interestingly, most of the schemes fit into general framework - regression or factor analysis. Indeed,

** Classical RRSQRT update step - Principal Component Analysis (PCA) with dimensionality trick;

** Truncation enhanced RRSQRT update step - Partial Least Squares (PLS) with random response vector, Lanczos bi-diagonalization procedure;

** Subspace Parsing Enhanced RRSQRT - Multivariate PLS, each subspace is ap-

proximated separately.

This observation leads to the producing different variations of the RRSQRT scheme, depending on the regression analysis approach adopted for the truncation step: regularized Least Squares, generalized PLS (gPLS), ridge regression, logistics regression, Hierarchical PCA, Multi-block PLS, Least angle regression etc.

Moreover, it is possible to combine truncation step and update step into one procedure using one of the factor analysis approaches, as it is done in the Reduced Rank Transform Square Root Kalman filter.

As an example, a Multi-block PLS regression scheme was used for the combined “truncation + update” step producing a breed of the RRSQRT filter. Filter performance is compared with the classical RRSQRT KF approach in application to the Advection-Dispersion equation for a multiphase pollutant.

References:

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