



## The use of conditioned probability density curves for hydropower planning at long, medium and short term time framework: A Colombian case study.

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An essential fact in hydrological design, forecasting and modelling is the assumption about the stationarity of probability density curve (PDC) that describes such variables as annual, monthly and daily runoff. Nowadays, every assessment related to the probabilistic characterization of hydrological regime fits theoretical distributions to empirical density curves obtained from observed data. This method however, is only valid for steady state conditions. Global climate change processes and the impacts on hydrological regime make steady state assumption unsupportive, where traditional hydrological assessment procedures need be refined in order to take account of changing driving factors. To this end conditioned PDC need be implemented by setting hydrological characterizations to an explain PDC evolution over the time. Such characterization can be done using the Markov process theory, where annual (monthly, daily) runoff ( $Q$ ) behaves as a Markov process, and can be described using conditioned probabilities  $p(Q_t|Q_{t-1}, Q_{t-2}, \dots, Q_{t-n})$ . Changing in such probability distributions are explained by a multidimensional Fokker Planck Kolmogorov equation of the form:

$$(1) \frac{\partial p(Q,t)}{\partial t} + \sum_{i=1}^n \frac{\partial [A(Q,t)_i p(Q,t)]}{\partial Q_i} - \sum_{i,j=1}^n \frac{\partial^2 [B(Q,t)_{ij} P(Q,t)]}{\partial Q_i \partial Q_j} = 0$$

Where:  $p(Q, t)$  is the two dimensional conditioned probability density curve of ran-

dom variable ( $Q$ ), and  $A(Q, t)_i$  y  $B(Q, t)_{ij}$  are the drift and diffusion coefficients, that define process behaviour and system's physical characteristics.

We derive an analytical approach to obtain  $A(Q, t)_i$  and  $B(Q, t)_{ij}$  based on an appropriate deterministic kernel, to simulate the PDC evolution as function of changes of driving factors, such as average rainfall shifting or land use. For annual and monthly runoff PDC forecasting a one dimensional variation of (1) can be set as follows:

$$(2) \frac{\partial P(Q, t)}{\partial t} + \frac{\partial [A(Q, t)P(Q, t)]}{\partial Q} - \frac{1}{2} \frac{\partial [B(Q, t)P(Q, t)]}{\partial Q} = 0$$

This partial differential equation is the solution for the stochastic rainfall – runoff equation proposed by Kovalenko (1993). In this paper we address the application of dynamic and pseudo stationary solutions for (2), for long, medium and short term hydropower planning. We present a hydrological probabilistic scenario for Colombia, resulting from climate change processes, as well a method to simulate PDC evolution for monthly affluence to hydropower reservoirs. All these results were include in the First Colombian Communication to the United Nations Framework for Climate Change.