Geophysical Research Abstracts, Vol. 8, 09990, 2006
SRef-ID: 1607-7962/gra/EGU06-A-09990

On the application of FFT and Wavelet Transform in gravity field modeling

A.A. Ardalan and R. Karimi

Department of Surveying and Geomatics Engineering, University of Tehran, Iran
(Ardalan@ut.ac.ir/Fax: ++982188008837)

The ability of Fast Fourier Transform (FFT) in gravity field modeling in terms of geopotential coefficients and capability of Wavelet Transform (WT) in removal of the observations errors from the results of the computations is studied and numerically tested. The surface spherical harmonic expansion can be written as follows:

$$f\left(\theta,\lambda\right)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\bar{P}_{nm}\left(\cos\theta\right)\left(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda\right) \tag{1}$$

where $\bar{P}_{nm}$ is the normalized Legendre functions of the first kind and $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are spherical harmonic coefficients. It can be easily shown that the above summation is equal to:

$$f\left(\theta,\lambda\right)=\sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\bar{P}_{nm}\left(\cos\theta\right)\left(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda\right) \tag{2}$$

or:

$$f\left(\theta,\lambda\right)=\sum_{m=0}^{\infty}A_m\left(\theta\right)\cos m\lambda+B_m\left(\theta\right)\sin m\lambda \tag{3}$$

where

$$\begin{gathered}A_m\left(\theta\right)=\sum_{n=m}^{\infty}\bar{P}_{nm}\left(\cos\theta\right)\bar{C}_{nm}\\ B_m\left(\theta\right)=\sum_{n=m}^{\infty}\bar{P}_{nm}\left(\cos\theta\right)\bar{S}_{nm}\end{gathered} \tag{4}$$

Equation (3) has the general form of a Fourier Transform (FT), therefore, $A_m(\theta)$ and $B_m(\theta)$ can be computed as Fourier coefficients as follows:

$$\left.\begin{array}{l} A_m(\theta) \\ B_m(\theta) \end{array}\right\} = \frac{1}{(1+\delta_{m0})\,\pi} \int_0^{2\pi} f(\theta,\lambda) \left\{\begin{array}{l} \cos m\lambda \\ \sin m\lambda \end{array}\right\} d\lambda \delta_{m0} = \left\{\begin{array}{l} 1, m=0 \\ 0, m \neq 0 \end{array}\right. \tag{5}$$

Having divided $A_m(\theta)$ and $B_m(\theta)$, the spherical harmonic coefficients $\bar{C}_{nm}$ and $\bar{S}_{nm}$ can be computed as follows:

$$\left.\begin{array}{l} \bar{C}_{nm} \\ \bar{S}_{nm} \end{array}\right\} = \frac{1+\delta_{m0}}{4} \int_0^{\pi} \left\{\begin{array}{l} A_m(\theta) \\ B_m(\theta) \end{array}\right\} \bar{P}_{nm}(\cos\theta)\sin\theta d\theta \tag{6}$$

Equation (3) is in the continues form and can be discritized for the practical application as follows:

$$\left.\begin{array}{l} A_m(\theta_i) \\ B_m(\theta_i) \end{array}\right\} = \frac{1}{L\,(1+\delta_{m0}+\delta_{mN})} \sum_{j=0}^{2N-1} f(\theta_i,\lambda_j) \left\{\begin{array}{l} \cos m\lambda_j \\ \sin m\lambda_j \end{array}\right. \tag{7}$$

where N corresponds to the number of meridian in the computations. To be able to verify the application of FT in computations of spherical harmonic coefficients a simulated problem is generated as follows: Using spherical harmonic coefficients of EGM96 the potential values are computed on a regular grid over the reference sphere $\mathbb{S}^2_{r=R}$, and by using Eq. (7) the coefficients $A_m(\theta)$ and $B_m(\theta)$ are computed and then using least squares method and Neumann weights, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are obtained and compared with the original coefficients. Besides, some noise is introduced into the potential values on the surface of reference sphere and WT is used to remove the effect of the noise. Following main results are obtained:

1. The accuracy of FFT without observation errors for generating of potential difference is $1.7245 \times 10^{-4}\frac{m^2}{s^2}$.

2. The accuracy of FFT with observation errors for generating of potential difference is $2.2695\frac{m^2}{s^2}$.

3. The accuracy of FFT implementing WT for removal of observation errors for generating of potential difference is $0.6624\frac{m^2}{s^2}$ which shows 70 percent improvement in the results.

4. WT with mother wavelet Bior6.8 is giving best results in the meridian direction, Coif4 in latitudinal direction and sym12 when we apply WT the whole observations vector.