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Compound Poisson-multifractal processes: a new mechanism for power law tails, an explanation for qD=3 in rain

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Many scaling phenomena are fundamentally pointlike but when averaged over larger enough scales are considered to be continuous and are mathematically modeled using fields (or densities of measures); examples range from the large scale distribution of matter in the universe, to seismic events to raindrops. We show how such systems can be modeled using compound Poisson/multifractal processes in which a multifractal particle number density field determines a Poisson probability density. If each particle has a "bare mass" (which need only have a thin tailed distribution), we show analytically that the effect of the "extra" Poisson variability is to create long power law tails on the distributions of the "dressed mass", i.e. the mass of a collection of many particles; the critical order of divergence of moments being determined by the spatial scaling of the governing multifractal densities. We apply this new model to rain; it is expected to be valid from huge (turbulence dominated) scales down to individual drop scales. The key is the liquid water density ? variance flux (?) which - following the HYDROP (stereophotography raindrop) observations and Corrsin-Obukhov passive scalar theory - is conserved from scale to scale it is the basic multifractal field. The link to the particle description is via the particle number density (n); we show how this can be determined from ? and the energy flux ?; we theoretically predict a k-2 spectrum for n which we confirm is close to observations. In order to perform simulations respecting these turbulence constraints we start with multifractal models of ? and ? cut-off by viscosity at the dissipation scale (roughly 1cm). From these fluxes we determine ? and n by fractional integration. At scales below 10cm or so, there is

typically only one drop in the corresponding sphere; we interpret n as the number density of a (compound) Poisson process and randomly determine the positions of the ith particle: ri. The masses mi, are determined from a unit exponential (Marrshall-Palmer) random variable Ei: mi=Ei ?(ri)/n(ri). The resulting measure (mi, ri) has the observed energy spectrum, the observed multifractal statistics (including the transition from particle scales to field scales) it also has realistic probability (fat tailed, power law) distributions for the dressed, (low resolution total mass in a large sphere) M. In this case, it predicts Pr(M’>M)?M-qD with qD=3 (this is an exact result coming from dimenionsional analysis). We show both on numerical simulations and on the HYDROP data that this prediction is accurately obeyed. In addition, it potentially explains numerious reports that qD?3 for the rain rate. Since it incorporates (in a highly inhomogeneous framework) the Marshall-Palmer exponential drop distribution as well as a Poisson particle process, it bridges the gap between classical and turbulence approaches. Numerical simulations spanning the range 1cm to 1000km can be readily produced. These simulations can be used for simulating radar reflectivity factors, effective radar reflectivity factors; extensions of the model can be used to simulate rain rates and rain gauges. These models can thus potentially solve various precipitation observer problems.