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Dynamically adaptive simulation of multiscale vortex interactions

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We present the Geophysical/Astrophysical Spectral-element Adaptive Refinement (GASpAR) code [4] and use it to simulate vortices in decaying 2D turbulence. The adaption criteria suggest techniques for vortex and coherent-structure identification. We will explore how the multiresolution hierarchical structure of spectral elements also suggests a way to quantify interactions among different vortices and the incoherent background flow.

Coherent-structure dynamics involves a large range of strongly interacting scales in space and time. *Fronts, plumes* and other structures in geophysical flows have been successfully simulated in recent years using dynamic adaptive refinement (DARe). There are a growing number of DARe codes and applications. All of these involve somehow partitioning the computational spatial domain $\mathbb{D}$ into disjoint elements $\bar{\mathbb{D}} = \bigcup_{k=1}^{K} \bar{\mathbb{E}}_k$. Almost all current DARe codes are based on finite-difference, finite-element or finite-volume spatial discretizations; i.e., a small number of values is used to represent the problem solution in each $\bar{\mathbb{E}}_k$. Thus *almost all current DARe simulations are intrinsically locally low-order* w.r.t. the size h_k of each $\mathbb{E}_k$. In contrast, there are a few DARe codes being developed that are locally *high-order* w.r.t. a parameter $\vec{p}_k$ in each $\mathbb{E}_k$. These *spectral-element methods* (SEMs) have a relatively long history in engineering but have only recently been applied to astro/geophysics, particularly as relates to detailed studies of small scales.

There are several properties of SEMs that make them very appropriate for complicated flow simulations. Perhaps most significant is the fact that *unlike low-order methods, SEMs are inherently minimally diffusive and dispersive*. This property is clearly important when trying to model flows at high Reynolds number Re (low viscosity) that characterize turbulent behavior. SEMs are spectrally convergent w.r.t. p_k when the solution is smooth in $\mathbb{E}_k$, but are also effective when the solution is not smooth elsewhere. Also SEMs can be used in high-resolution studies of turbulence in domains with complicated boundaries. SEMs also are naturally parallelizable (e.g., [3]), which is important for modeling high-Re flow with many degrees of freedom (d.o.f.) involving multiple spatial and temporal scales.

SEM-DARe is enhanced w.r.t. low-order DARe in regard to *adaptivity criterion*. Since every $\mathbb{E}_k$ contains $\prod_{\alpha=1}^{d}(p_k^\alpha + 1)$ local d.o.f., that information provides a local accuracy estimate. Possibilities that have been tested include estimating local Legendre-spectrum decay, or comparing relative contributions to the $\vec{u}$-norm in $\mathbb{L}^2(\mathbb{E}_k)$ from $\mathbb{E}_k$ vs from its 2^d children $\mathbb{E}_{k'}$ by d-way bisection. The latter possibility is enabled by the multiresolution hierarchy intrinsic to SEM-DARe, i.e.,

$$\operatorname*{span}_{\vec{\jmath}} \phi_{\vec{\jmath},k} =: \mathbb{P}^k \subsetneq \bigcup_{\bigcup_{k'=2^d k+1}^{2^d(k+1)} \mathbb{E}_{k'} = \mathbb{E}_k} \mathbb{P}^{k'} \quad \Rightarrow \quad \bigcup_{k=(2^{\ell d}-1)/(2^d-1)}^{(2^{(\ell+1)d}-1)/(2^d-1)-1} \mathbb{P}^k =: \mathbb{V}^\ell \subsetneq \mathbb{V}^{\ell+1},$$

as introduced in detail by [1]. The projection operator $\mathrm{proj}_{\mathbb{V}^\ell \setminus \mathbb{V}^{\ell-1}}$ on fine scales is the continuous spectral-element analog to the discontinuous spectral-element projection used by [2], but is not orthonormal; nevertheless, it allows exact energy decomposition. These projections are rigorous analogs for spectral elements, of ideal Fourier spectral wavenumber bandpass filtering on uniform grids.

References

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