



Benchmarking of structured and unstructured mesh approaches in large material contrast Stokes problems

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Geomaterials are generally heterogeneous on many scales and characterized by large material property contrasts. The direct numerical resolution of the behaviour of such materials represents, despite the advances in modern computer technology, a formidable computational challenge. Using the analytical solution of Eshelby for the elastic fields in and around an ellipsoidal inhomogeneity adopted for an incompressible Newtonian Stokes flow we illustrate benchmark comparisons of structured and unstructured FEM analysis. The solution of heterogeneous three-dimensional problems requires large numbers of degrees of freedom to resolve geometry and mechanics accurately. For such problems iterative methods prove to be attractive compared to direct solvers due to their low memory requirements and to a higher potential of parallelism. However, their sensitivity to matrix conditioning underlines the importance of the preconditioning stage. The family of approximate factorizations (XIC) used as preconditioners with high-performance PCG solvers turn out to be a powerful solution strategy. A Stieltjes matrix, determined from the original stiffness matrix via DC reduction scheme, serves as an input to the approximate factorization procedure. An efficient Uzawa algorithm for the solution of the 3D steady-state Stokes problem with 15-node $P_2^+ - P_1$ Crouzeix-Raviart, discontinuous-pressure, tetrahedral elements is utilized. This unstructured method is compared to methods with structured Cartesian grids and potentially non-uniform spacing. This structured grid strategy allows for the application of rapid solution techniques such as ADI, spectral methods, cyclic reduction, and multi-grid. Both, structured and unstructured, approaches have distinct advantages and fields of applicability. For example ADI type methods enable us to calculate in the order of one billion DOFs in 3D, while unstructured methods allow for mesh-adaptation to the specific geometry and solution. Particular benchmarks that we present include speed, memory efficiency, max. problem solvable, and accuracy

(compared to true analytical solution).