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Stability of dispersive long wave solvers in variable depth

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In the applied numerical tradition stability analysis of wave models has been extensively carried out by the Von Neumann method where propagation of linear wave modes in a uniform medium is studied. The requirement of uniform medium then also implies a uniform grid resolution. Often this method leads to simple stability criteria of the CFL type, with the interpretation that the signal speed of the discrete media (grid) may not be smaller than the physical wave speed. Such criteria is then often, and in many circumstances successfully, conveyed to more complex situations, where ad-hoc modifications are made to CFL type criteria to include nonlinear effects and variable coefficients. However, it is generally recognized that nonlinearity may give rise to instabilities and spurious phenomena that is not contained in the van Neumann analysis. An important example is the effect of aliasing. Hence, even if most nonlinear models evade rigorous analysis there is an awareness concerning their performance that urge for some measure of testing and careful investigation of results. Still, in physical sciences, like ocean modeling, there is a push for application of standard, and even commercial, models that seems to weaken the critical sense of modelers.

While nonlinear aspects of numerical modeling are appreciated, spurious effects related to variable coefficients (depth or grid) have traditionally received little attention. Nevertheless, investigations that have been carried out indicate that such problems do exist. For instance, an eigenvalue analysis of the propagation matrices of one of the standard models for ocean waves revealed that the Coriolis effect, in combination with variable depth, may lead to instabilities in form of exponentially growing modes (Espelid et al. 2000).

In this presentation we address linear dispersive models, of the Boussinesq type, in variable depth. As compared to the LSW equations the dispersive counterparts display a much more complicated form, also concerning the appearance of depth gradients. The topic of weakly dispersive long wave models has undergone substantial development during the last two decades, resulting in a variety of proposed forms for numerical modeling and a markedly smaller sample that have been put to work in existing models. Meanwhile, Boussinesq type equations have been applied to increasingly demanding problems related to tsunamis and coastal engineering in general. Hence, the robustness of such models in realistic contexts, including roughly digitized bathymetries, should be examined carefully. Herein, we study a selection of Boussinesq type models, including the popular formulation of Nwogu (1993) that later has been developed further by others (Wei et al. 1995, Hsiao et al. 2002). Employing eigenvalue analysis in closed and infinite basins, with simplified geometries as sea mounts and shelves, we indeed discover that instabilities, of a more or less severe nature, may arise. Moreover, in some cases it is demonstrated that the instabilities render the equation set in question useless as a general tool for modeling of ocean waves, like tsunamis.