



Differential geometry of nonlinear dynamical systems: Rikitake dynamo system and Lorentz model

T. Yajima and H. Nagahama

Department of Geoenvironmental Sciences, Graduate School of Science, Tohoku University
Aoba-ku, Sendai 980-8578, Japan (yajima@dges.tohoku.ac.jp/Fax: +81-22-795-6634)

Differential geometrical method, KCC-theory, is useful for investigating a behavior of nonlinear systems in geomagnetism and meteorology. We study Rikitake dynamo system as nonlinear dynamical system in geomagnetism. Rikitake dynamo system is governed by 2nd order differential equations in electrical and mechanical system. The geometrical theory of 2nd order differential equations is often called KCC-theory. Following the KCC-theory, we calculate some geometric objects. The connection means interactions between electrical and mechanical system. The interactions are represented by the mutual inductances which express the coupling of each eddy of electric current and fluid motion in the earth's core. The torsion tensor which represents the "discrepancy" from the electrical system and mechanical system consists of mutual inductance and resistance. In the large torsion case, the interaction of eddies of electric current and fluid motion is strong and the conductivity of fluid is small. The discrepancy of trajectory is wide in phase space. So, the geomagnetic field reverses aperiodically during the short period as Jurassic period. On the other hand, when the components of torsion tensor are small, the interaction of eddies is weak in the large conductivity. In this case, the geomagnetic field does not reverse during the long period as Cretaceous Superchron. So, the torsion expresses the stability of geomagnetic reversal. Then, we obtain the curvature tensor. The eigenvalue of curvature tensor gives the parallelism of trajectories. The difference of eigenvalues represents the change of mode of trajectories in time series. The behavior of magnetic field in non-reversal field is determined by the curvature tensor. Moreover, the Lorentz model in meteorology mathematically belongs to the same type of differential equation of Rikitake dynamo system. So, the geometrical properties of Rikitake dynamo system and Lorentz model can be analyzed in terms of KCC-theory.