



## Nonlinear quasi-geostrophic balance

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In an effort to better understand the dynamical behaviour of the Earth's atmosphere and oceans, many researchers have sought simplifying concepts which are fundamental to these flows. One such concept which applies to rotating stratified geophysical flows is that of "balance" (see Ford *et al* 2000). Put simply a balanced flow is one where the dynamics of the flow are controlled solely by the "potential vorticity" (PV), a materially conserved scalar quantity. Balance holds to a high degree for intermediate to large scale fluid motions. There have been many previous models which exploit balance to derive equations of motion (Bokhove 1997; Muraki *et al* 1999). Here we introduce a new way to diagnose the balanced dynamics. The basis of this model is the reformulation of the non-hydrostatic Boussinesq equations introduced by Dritschel & Viúdez (2003), wherein the PV and the two horizontal components of the ageostrophic vorticity are used as prognostic variables. This has an advantage over using more traditional variables such as velocity and pressure as it exposes the separation between the balanced and imbalanced part of the flow with the PV representing the balanced part of the flow and the ageostrophic vorticity effectively being the departure from thermal wind balance. In the limit of small Rossby number these equations reduce to the quasi-geostrophic equations. At this order the primary fields are of order Rossby number.

In our model we use scaling analysis to obtain the next order corrections to the quasi-geostrophic equations. This involves removing the time dependence of the ageostrophic vorticity components. These higher order corrections, which we refer to as the "nonlinear quasi-geostrophic" (NQG) equations, depend only on the first order fields which are obtained from the PV. This process is repeated iteratively to obtain the higher order fields to a prescribed accuracy. Effectively this iterative method is a diagnostic method to obtain the balanced fields up to order Rossby number squared from the full PV. This approach is similar to the  $QG^{+1}$  model of Muraki *et al* 1999, except our approach applies to the full non-hydrostatic Boussinesq equations. Also we apply the iterative method to obtain a more accurate approximation of the balanced fields. Here balance is diagnosed in turbulent flows using the NQG balance method and another technique (Viúdez & Dritschel 2004). Overall the NQG method captures a significant portion of the balanced dynamics.

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