



## **A localization approach to estimating potential fields from noisy, incomplete data at satellite altitude**

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We show that the estimation of potential fields from noisy, incomplete data taken at satellite altitude is a problem of spatio-spectral localization. We have derived a new basis of spherical Slepian functions that are optimally concentrated inside of arbitrarily shaped close domains on the sphere. These are calculated by matrix diagonalization of a localization kernel. Constructing the spatio-spectral localization kernel and diagonalizing it is computationally cumbersome and often numerically unstable. A special case arises when the concentration region is a circularly symmetric polar cap, a pair of antipodal caps, or their complement, a latitudinal belt, as with the geodetic polar gap. In that case, the operator that bandlimits a field on the unit sphere and projects it onto the polar caps commutes with a Sturm-Liouville operator. Its eigenfunctions can be computed extremely accurately and efficiently by diagonalizing a tridiagonal matrix with analytically prescribed elements. The gains in ease, speed, and accuracy thus achieved will make the usage of Slepian functions in earth and planetary geodesy practical, as we show by example.

The solution to the geodetic estimation problem can be found either by a damped least-squares approach in the spherical harmonic domain (SH), or by expanding the solution in a truncated Slepian function basis (SG). The SG approach uses fewer basis functions, and is thus more compact than the SH case. It is also more intuitive, as, in the expression for the mean square estimation error, it neatly divides the variance and the bias over the region of coverage and lack of coverage. The damped spherical harmonic approach achieves similar results but at a higher computational cost, and without the intuitive division over the measured and unmeasured areas characteristic of the Slepian approach.