



On the scale-space markov approximation for efficient ensemble filtering

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The effectiveness of a data-assimilation method depends on its ability to address the issues of nonlinearity, uncertainty and dimensionality. Ensemble filters use sampled states to represent uncertainty, and have been shown to be effective for a variety of nonlinear dynamical systems.

Several variants of the ensemble Kalman filter have been proposed. One of them uses square-root formulations with localization. Localization serves to remove spurious long-range correlations and yields added benefit for large estimation problems. However, the effectiveness of localization is clearly dependent on the extent of spatial support which, in realistic scenarios, is state dependent. Another approach is based on a wavelet decomposition of the state, where updates are limited only to lower sub-bands of the wavelet decomposition. This method also tackles the dimensionality issue, and can filter spurious correlations. However, energy in the higher-frequencies is not accounted for; a disadvantage that is shared with other spectral truncation methods. More recently, a “multi-scale ensemble Kalman filter” has been proposed. This variant constructs a multi-resolution tree from the ensemble with a “forward-backward” sweep through the tree for inference. This algorithm can significantly speed up estimation time, and also seems to attenuate spurious correlations. But its weakness shows up as boundary artifacts introducing artificial gradients at cell boundaries that cannot dynamically exist.

We introduce a new variant of ensemble filtering that avoids the problems inherent in localization, wavelets or trees, and filters large state-spaces efficiently. This method combines approximations in scale, space and spectral rank of the information topology.

The central idea is to decompose the domain into a lattice of nodes extending in space

(the state's dimensions), and scale (multi-resolution). Once this lattice is identified using the ensemble, each node implements a local ensemble filter on a very small subspace. This ensemble filter can be implemented using the (ensemble) Kalman or particle filter. Additionally nodes can send messages to adjacent nodes in the scale-space lattice and, in effect, produce a spatially smooth and artifact-free estimate.

In practice, spatial-smoothing is made optimally efficient using a outward-inward sweep from the interior of the lattice to its boundaries and back in the same way as this can be done for smoothing markov chains in time. We therefore adapt a previously developed ensemble smoothing algorithm that was shown to be the most efficient ensemble smoother at this session last year.

The net result is a highly parallelized filter, where the spatial range of correlations can be controlled in an artifact-free manner. We demonstrate its effectiveness using experiments on the MIT-GCM in an unstable flow regime.