



## **Nonlinear, data-based reduced models for climate variability**

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We present a novel methodology for constructing a hierarchy of empirical, stochastically forced models for the analysis and simulation of climate variability. This methodology encompasses both linear and nonlinear time-dependent models that aim to best describe the data set's statistics. We generalize the so-called linear inverse model (LIM) approach, which is restricted a priori to linear dynamics with additive forcing by noise that is white in time; in LIMs, multiple linear regression (MLR) is used to estimate the linear deterministic propagator of the dynamics, as well as the spatial structure of the stochastic forcing, directly from the observations.

Our empirical mode reduction (EMR) methodology generalizes LIMs in two ways: (i) MLR is replaced by multiple polynomial regression (MPR) to allow the deterministic propagator to be quadratic or cubic; and (ii) the single level of additive stochastic forcing is replaced by the possibility of multiple levels of such forcing. In this multi-level generalization, the residual stochastic forcing at a given level is subsequently modeled as a function of variables at this, and all preceding levels. The optimal number  $L$  of levels is determined from the data so that the lag-0 covariance of the residual forcing converges to a constant matrix, while its lag-1 covariance vanishes. When  $L$  is larger than unity, the forcing at the first level will be “warm-colored” (i.e., not white).

We apply this methodology to study low-frequency variability of the Northern Hemisphere (NH) atmosphere. First, we consider an observational data set based on 44 winters of 700-hPa geopotential heights over the NH. The data are projected onto empirical orthogonal functions (EOFs) and the leading nine EOFs provide an optimal

predictor set. With these predictors, a three-level, quadratic inverse model produces the best fit to the observed statistics. This fit is verified by checking the probability density functions (PDFs), as well as the singular spectra of the NH data set and those simulated by the reduced model. Both data and model PDFs reproduce the minimal set of three flow regimes of Cheng and Wallace (1993, JAS) and Smyth et al. (1999, JAS)

Second, we analyze the output of a long simulation of a global quasi-geostrophic, three-level (QG3) model with topography. The reduced model has 45 variables and captures well the non-Gaussian features of the QG3 model's PDF. An analytical and numerical study of the reduced model starts with the fixed points and oscillatory eigenmodes of the model's deterministic part and uses systematically an increasing noise parameter to connect these with the behavior of the full, stochastically forced model version. The results of this study point to the origin of the QG3 model's multiple regimes and intraseasonal oscillations and identify the connections between the two types of behavior.