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A new look at an old problem: The 'Sandström theorem'

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The 'Sandström theorem' as interpreted by Jeffreys (1925) is that in a flow maintained by a temperature difference the pathline from the cold region to the warm region must lie below the return path. Although this argument has been formulated ca. eighty years ago and could be useful for the interpretation of geophysical flows, it seems to have been largely ignored - the paper of Jeffreys being known as a criticism of an ealier statement of the theorem but not for the reinterpretation of the theorem. Here the Jeffreys argument will be examined in some detail. A formal demonstration of it for a rotating fluid requires three assumptions about the relative circulation around a closed material line: (i) the flow is steady, (ii) the closed material line is a closed streamline, and (iii) the work done by friction along the streamline is negative. The argument extends to unsteady flows, thereby relaxing (i-ii), if the absolute circulation along the material line is a bounded function of time - a condition that is easily shown to be met for flows with small Rossby number. Its validity for time-periodic 2D flows of horizontal convection is verified numerically. Poincaré sections reveal the presence of chaotic particle transport in these flows, even though the Eulerian velocity fields have a simple time-dependence. In spite of chaotic advection, particle motion is in general downwards in the cold region and upwards in the warm region of the fluid, which is consistent the flow shape envisioned by Jeffreys. Our results give support to the validity of his argument for the unsteady case, thereby enhancing its relevance for the interpretation of the basic structure of geophysical flows.