



Methods for characterizing long-range persistence

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Many time series in the Earth Sciences have been shown to exhibit persistence (memory) where large values (small values) 'cluster' together. Here we examine long-range persistence, where one value is correlated with all others in the time series (vs. short-range persistence). A time series is long-range persistent (a self-affine fractal) if $S \propto f^{-\beta}$; with S the power-spectral density, f the frequency and β a constant that measures the long-range persistence 'strength'. We compare five common analysis techniques for quantifying long-range persistence: (a) Power-spectral, (b) Wavelet variance, (c) Detrended Fluctuation, (d) Semivariogram, and (e) Rescaled-Range (R/S) analysis. To compare each technique, we construct 18,000 synthetic fractional noises with different persistence strengths (β), both normal and log-normal distributed. With respect to the performance of the different methods, we find: (a) Power-spectral and wavelet analyses are the most robust for measuring long-range persistence across all β for both normal and log-normal noises, although 'antipersistence' is over-estimated for non-Gaussian time series. The errors (95% confidence) get larger as the length N of the time series decreases, as $N^{-0.5}$. (b) Detrended Fluctuation Analysis is appropriate for signals with long-range persistence strength $-0.2 < \beta < 2.8$, and has very large 95% confidence intervals for non-Gaussian signals. This technique is commonly used in the climatology and biomedical communities, and care should be taken when using it to quantify long-range persistence for non-Gaussian time series. (c) Semivariograms are appropriate for signals with long-range persistence strength $1.0 < \beta < 2.8$; it has large 95% confidence intervals and systematically underestimates log-normal noises in this range. (d) Rescaled-Range Analysis is only accurate at $\beta \approx 0.7$, and systematically under- or over-estimates for other values. We conclude that some techniques are much better suited than others for quantifying long-range persistence, and the resultant β (and associated error bars on them) are sensitive to the one-point probability distribution, the length of the time series, and the technique used.